

# A 'Sustainability Window' of Urban Form

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## Abstract

With global environmental change and the rise of global megacities, environmental and social externalities of urban systems, and especially of urban form, become increasingly prevalent. The question of optimal urban form has been debated and investigated by different disciplines in numerous contexts, including those of transport costs, land consumption and congestion. Here we elucidate theoretically how urban form and the urban transport system systematically modifies sustainability concerns, such as greenhouse gas emissions, local air pollution and congestion. We illustrate our analytical considerations with empirical analysis. Denser urban form would almost unambiguously mitigate climate change, but it would also lead to undesired effects, such as a higher proportion of urban dwellers affected by air pollution. Our study presents a 'sustainability window' by highlighting trade-offs between these sustainability concerns as a function of urban form. Only a combination of transportation policies, infrastructure investments and progressive public finance enables the development of cities that perform well in several sustainability dimensions. We estimate that a residential population density between 50 and 150 persons/ha and a modal share of environmental modes above at least 50% corresponds to the sustainability window of urban form. The parameters of the sustainability window of urban form is subject to policy changes and technological progress.

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## 1. Introduction

The sustainability of cities challenges scientific constituencies that emphasize issues participation [1], or global environmental governance [2], or the transportation system [3]. By 2050, about

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67% of the world's population is expected to live in cities – a total of 6.3 billion urban dwellers  
5 in a world population of 9.3 billion people [4]. Issues of urbanization and global environmental  
change, local environmental challenges along with equity concerns become increasingly prevalent.  
Catching perhaps the highest attention, cities also enter the spotlight as places combatting climate  
change. Inversely, climate change literature focuses progressively on cities, revealing a significant  
local mitigation and adaptation potential in an age of rapid expansion of urban areas [5]. About 40%  
10 of all transport emissions occur in urban areas [6], and therefore CO<sub>2</sub> reduction efforts increasingly  
focus on this mitigation potential [7, 8].

The key issue underlying the city-transport-climate nexus is, arguably, urban form. It has long  
been argued that higher urban density translates into lower per capita transport energy consumption  
[9], a relationship which also emerges straight from theory, specifically from the canonical framework  
15 of urban economics [10, 11]. Detailed analysis elucidates that this relationship can in fact be  
explained by more specific urban form indicators, such as 'distance to work' and 'connectivity' [12].  
Nonetheless, urban population density remains a reasonable proxy for energy use in transportation.

Clearly, urban modifications are subject to multiple objectives. Urban transport - especially  
individual motorized transport - causes manifold environmental and social externalities that go  
20 beyond the climate change externality. Air pollution from vehicle exhaust constitutes one of the  
most serious public health hazards in cities, and congestion is an economic externality with high  
costs. For example, in emerging economies like China, urban transport, air pollution and congestion  
are perceived as much stronger concerns than climate change [13] and in the 1980s, the European  
environmental debate focused on acid rain as one of the most negative consequences of transport.  
25 However, the magnitudes of all of these externalities change with urban density [14, 13, 15]. Im-  
portantly, while energy use reduces with increasing urban density, air pollution and costs of living  
become increasing burdens for residents. Hence, the local and global rationales for policies influ-  
encing urban form indicate a considerable trade-off between the benefits and harms of increasing  
urban density. The increasing pervasiveness of the climate change mitigation challenge, along with  
30 local environmental and social urban issues calls for a fundamental overhaul of urban mobility. This  
debate is spanning from the climate change community over urban planners, architects and also  
local initiatives who think about the wider goal of 'sustainable cities' and liveable urban centers  
[16]. The debate hence needs to tackle many issues simultaneously.

Urban density alone, however, cannot explain urban transport energy use. Modal shares also

35 influence urban energy use and emissions. Higher modal shares of public transit, cycling and walking – i.e. lower modal shares of car use – typically result in lower greenhouse gas emissions in urban transport systems [7, 17]. Urban form and modal shares are co-dependent on each other. A minimum urban density is required to enable a shift towards low-carbon modes and, in fact, increases financial viability of public transport systems [11]. Sprawling cities with long distances  
40 such as Los Angeles or Houston encounter tough challenges to enable bike commuting, or to get sufficient ridership numbers for public transport systems in suburban areas. Choosing the right infrastructure design plays another important role for enabling modal shift to cycling. This been demonstrated by the city of Copenhagen, which has driven a massive modal shift to cycling by providing the pertinent infrastructure for its citizens, and also improved urban quality of living [18]  
45 [16]. Besides urban energy use, modal shares also have relevant impacts on congestion, air pollution and urban quality of life. Hence, there is a strong rationale to include modal shares as a co-factor of urban form when investigating the trade-offs of urban form and the environment.

Here we contribute to this challenge by systematically analyzing how urban density and modal share modify global and local environmental and social benefits. We are basing this study on a  
50 straightforward monocentric urban economics model. This allows us to conceptualize trade-offs between the individual dimensions and to identify a ‘sustainability window’ of urban form where all of these concerns are adequately addressed. The choice of a monocentric city model means that the study stays conceptual, and that we do not account for policentricity, technological change or other more complex issues that lie in the very nature of urban studies. We conclude by indicating  
55 policy options that can modify the sustainability window to further improve the benefits in at least some dimensions.

We proceed as follows. In section 2, we explain how urban form modifies climate change, congestion, air pollution and urban land rent. We provide an analytical model that predicts how transport costs and modal shares influence these sustainability concerns, and how changes in urban form can  
60 realize concurring benefits. In section 3, we present the results of this conceptual analytical model and the emerging ‘sustainability window’. We also provide examples of policy options that improve the performance of the sustainability window. In section 4, we statistically analyze the factors contributing to the sustainability window. Section 5 discusses policy implications and concludes the article.

## 65 **2. Urban form, urban transport and its co-benefits**

The quality of urban life depends on multiple dimensions and on the local context, but it seems clear that it is virtually always influenced by a few important dimensions. At the heart of our analysis lie the sustainability concerns of urban transport and urban form. A thorough and general description of urban transport externalities is given by [14], for example. In the following, we first describe the classic urban economics framework upon which our study is built. It is used to compute density profiles for several types of cities, differing from each other through their modal shares and urban densities. The key control variables which influence the externalities of urban transport are the generalized transport price and a parameter modifying the share of individual motorised transport. Section 2.2 follows by describing the implementation of each sustainability concern, or in economic parlance, externality, that can be attributed to car transport: air pollution, road congestion and climate change. A social concern to represent variability of land rents with density is also described.

### *2.1. Modelling urban form and the dimensions of urban transport*

We have chosen to use a straightforward monocentric city model (see description below) and thus build on the well-founded framework of urban economics. It is straightforward to implement the sustainability concerns and conceptually present the influence of urban density and modal share. We briefly want to shed light on the extensive science of urban modelling, which offers much more advanced approaches. A concise overview is given by [19], who, apart from classic urban economic models, highlights the philosophy of model-building and categorizes the most relevant urban modelling disciplines into the land-use transportation community and those researchers relying on agent-based or cellular automata models. Agent-based models (an extension of cellular automata) join into complexity science and are increasingly used for urban planning, partly because of their aptitude to capture dynamic processes [20, 21]. Land-use transportation (LUT) models are based on the basic theory of urban and regional economics and include a multitude of variables and information from a specific region. Hence, they tend to be case-specific and very well suited e.g. for regional case studies, see e.g. [22]. In fact, [23] have employed LUT models to capture trade-offs of urban compaction under sustainability dimensions, albeit focused on case-studies. We now present a conceptual framework for a more generalized analysis.

2.1.1. *The classic urban economics framework*

The study builds on the classic urban economics model as introduced by Alonso, Muth and Mills, hence the model is referenced to as the *AMM model*. It is thoroughly explored by [10]. This theory assumes a monocentric city, in which residents commute into the city centre. All residents share the same income  $Y$  and the same utility function. Income  $Y$  is spent on commuting costs  $T(r)$ , rent  $R(r)s(r)$  and bread consumption  $z$ .

$$Y = z + R(r)s(r) + T(r)$$

95 Residents maximise their utility, which is assumed a log-linear Cobb-Douglas function.

$$U(z, s) = \alpha \log(z) + \beta \log(s) \quad \text{with} \quad \max U(z, s) \quad (1)$$

In the case of constant urban population (closed city case) and absentee landownership, bid rent  $\psi(r)$ , dwelling space  $s(r)$  and population density  $\rho(r)$  are described by [10]:

$$\Psi(r, u) = \alpha^{\alpha/\beta} \beta (Y - T(r))^{1/\beta} e^{-u/\beta} := R(r) \quad (2)$$

$$\rho(r, u) = \left[ \alpha^{-\alpha/\beta} (Y - T(r))^{-\alpha/\beta} e^{u/\beta} \right]^{-1} \quad (3)$$

Bid-rent  $\psi(r, u)$  is assumed to equal per-area rent  $R(r, u)$ . The urban population density  $\rho(r, u)$  is the central parameter to describe urban form. In this model, utility  $u$  is constant over  $r$  and can therefore be dropped. Total population  $N$  is normalised to  $N = 1$ :

$$N = \int_0^{r_c} \rho(r) \, dr = 1$$

2.1.2. *Extension: Modelling different modal shares*

We assume that a part of the population uses public transport (comprising non-motorized transport), and the other part uses car transport. Therefore, we arithmetically decompose the density profile provided by equation 3 into these two parts by applying

$$\rho(r) := [\zeta_{pt}(r) + \zeta_{car}(r)] \rho(r)$$

where

$$\zeta_{pt}(r) + \zeta_{car}(r) = 1 \quad \forall r \in [0, r_c]$$

The public transport density profile  $\zeta_{pt}$  is created by applying a logistic function to the generic density profile.

$$\zeta_{pt}(r) = 0.8 (1 + e^A)^{-1} \rho(r)$$

100 with

$$A = \frac{-\kappa\rho(r)}{\rho_{max} + \gamma} \quad (4)$$

In equation (4), parameters  $\kappa$  and  $\gamma$  are scaling parameters influencing the shape of the logistics function. In the following model runs,  $\kappa$  is held constant, while  $\gamma$  is varied to create a range of different mixes of modal shares, as is illustrated in Fig. (A.6). The per-distance price of the public transport mode,  $m_{pt}$ , is assumed to be lower than the car-commuting price  $m$ . Time costs and  
 105 similar factors are not accounted for. This leads to a utility increase for public transport users. This is discussed in further detail 2.2.4 and in Appendix B.

We emphasize that the existence of a public transport price  $m_{pt} \neq m$  does not feed back into the economic model, as it is done in [11]. We have chosen this non-dynamic approach because the main purpose of this paper lies in comparing the externalities and impacts of urban car transportation  
 110 and setting up of a model to simulate different city types. A dynamic model with feedback of externalities could be a possible future extension of this study.

The public transport modal share equals the total fraction of public transport users over the complete number of urban residents, integrated from the city centre at  $r = 0$  to the outer city limit at  $r = r_c$ . The total number of residents is normalised and we have:

$$\eta_{pt} = \frac{\int_0^{r_c} \rho_{pt}(r) \, dr}{\int_0^{r_c} \rho(r) \, dr} = \int_0^{r_c} \rho_{pt}(r) \, dr$$

Then, the modal share of car transport is

$$\eta_{car} = 1 - \eta_{pt}$$

### 2.1.3. Numerical Implementation

The Alonso-Muth Mills model is implemented numerically. The routine iterates utility  $u$  to  
 115 determine a density profile for a given population number  $N = 1$  and a given transport cost  $m$  in the standard economic model described above. The utility obtained equals the analytical utility from equation 1. The code, written in the Python language, is contained in Appendix C.

## 2.2. Sustainability concerns

Here we describe four sustainability concerns and how we implement them into the model. We regard  $CO_2$  emissions – associated with a climate change impact – and *local air pollution* as environmental dimensions, with the latter also playing a strong role in public health concerns. We see *congestion* primarily as an economic concern, and *cost-of-living* as a social concern.

### 2.2.1. Greenhouse gas emissions

$CO_2$  emissions from urban transport contribute to anthropogenic climate change [24]. The International Energy Association (IEA) estimates that urban areas contribute 71% of global greenhouse gas emissions [6], and the Global Energy Assessment (GEA) reports a share between 53% and 87% [25].

Crucially, denser cities dramatically reduce urban transport distance traveled, transport energy use and associated  $CO_2$  emissions [9]. Detailed analysis elucidates that this relationship in fact can be explained by more specific urban form indicators, such as 'distance to work' and 'connectivity' [12]. In addition, population density appears as a highly significant factor in a comparison of transport GHG emissions of global cities, but much less so in a comparison of cities within the same continent [26]. A sophisticated statistical treatment (by hierarchical threshold regression) of worldwide cities highlights the importance of urban form, but suggests an interpretation in the context of economic development, income growth, and regional fuel prices [27]. In addition, urban form influences not only transport emissions but also residential household emissions. For example, a non-linear econometric analysis demonstrates that urban population density is an important classifier of emission types of human settlements, and that its importance as drivers of emissions depends on the socio-economic context [28].

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The emission of climate-change causing  $CO_2$  is assumed to be proportional to transport distance. Integrated over distance travelled as a function of marginal transport costs, and with everything else constant (see [11]), we specify the climate change externality as

$$C_{CO_2} = b_{CO_2} \int_0^{r_c} r \rho_{car}(r) dr$$

with  $C_{CO_2}$  specifying the external effects of climate change, and  $b_{CO_2}$  as a scaling factor.  $r$  is the distance from the city centre, the integration boundaries are the city centre at  $r = 0$ , and the outer city limit at  $r = r_c$ .

### 2.2.2. Air pollution

145 Urban air pollution has always been a disturbing hazard in industrial cities. Emblematically, the Great London Smog caused more than 12,000 premature deaths in 1952 [29]. Its current extent becomes most visible in Chinese megacities, where air pollution causes millions of premature death [30] and even impacts the distant Californian coast [31]. There is extensive research on air pollution modelling, ranging from street-level scale studies (see the review by [32]) over applied models for 150 operational forecasting [33] up to regional and continental-scale air pollution transport modelling studies [34]. Models have introduced crucial parameters, such as the essential role that trees play in removing air pollution from cities [35]. One attempt to describe the impact of air pollution for housing choice in urban economics has been presented by [36]. As a social cost factor motivating urban and regional policy making, air pollution often outpaces climate change [37, 38, 39]. Air 155 pollution impact depends on the scale and the nature of emissions, as well as on local climate and on urban form. The latter aspect is of particular relevance for our analysis.

Generally, air pollution from internal combustion engines comprises several gases and aerosols, most particularly  $\text{NO}_x$ ,  $\text{SO}_2$ ,  $\text{O}_3$  or particulate matter of a specific size such as  $\text{PM}_{2.5}$  (the weight concentration of particles of an equivalent diameter smaller than  $2.5 \mu\text{m}$ ). In the remainder of this 160 paper, we use  $\text{PM}_{2.5}$  as a proxy for air pollution. In contrast to greenhouse gas emissions, it is the concentration of pollutants that is important, and more specifically, the *intake volume* [40, 41]. Intake volume specifies the level of pollutants breathed in by humans and therefore depends on population density  $\rho(r)$  at location of emission. In other words, intake fraction increases with the number of receiving individuals who are close to the emission source. Pollutants from cars are 165 emitted in the streets, where individuals walk. This makes cars much more relevant for urban air pollution than, for example, industrial plants. If advection is considered as the dominant removal process of pollutants, the intake fraction scales with population over the square root of city area [40]. We make use of this relationship in our model: the emission of pollutants is assumed to be proportional to through traffic, which can be specified as the number of people living further away 170 from the city center than the location under consideration.

$$C_{AP} = b_{AP} \int_0^{r_c} \rho(r) \Omega(r) dr \quad (5)$$

$$\Omega(r) = \int_r^{r_c} \rho_{car}(r) dr \quad (6)$$



Equations (5) and (6) are useful approximations to link air pollution to urban form. However, they take into account neither meteorological parameters nor pollution removal processes, technological factors or other properties of urban form. We are aware of these shortcomings, but believe that a simple approach helps convey the basic message in the following sections.

### 2.2.3. Congestion

Congestion is an extensively studied externality of urban transport. It is most prevalent in expanded cities, although decentralization reduces its impact [42]. The economic cost of congestion is often assumed to be the sum of the marginal cost of driving plus the additional travel time spent due to congestion. Congestion is dynamic and non-linear with traffic. Physically, it is described in different forms such as *bottleneck congestion* or *flow congestion*. So-called *textithypercongestion* is the most severe consequence of congestion, when traffic comes to a standstill [14]. Many infrastructure measures, such as the creation of efficient public transport or support of bike traffic can stimulate a shift of travellers to other modes of travel and therefore relieve streets from cars. Congestion can be reduced with congestion charging, which can stimulate a modal shift or departure time shift. London, Singapore, or Stockholm are important examples. The study of optimal congestion charging has produced numerous analyses and estimations of the cost of congestion. Given that one way to look at congestion is working time lost, the cost of congestion is proportional to GDP. For the London congestion charge, the 'ROCOL' working group [43] estimates a congestion cost of 15.6 Euros per hour. For a possible Paris congestion zone, lower values were used [44]. In Beijing alone, the annual cost of congestion is estimated at the equivalent of 3 billion Euros per year by [13]. In the urban economics literature, numerous studies focus on commuters' endogenous departure time choices, the social and personal costs of congestion and effects from congestion charging, such as [45], [46], [47] or, more recently, [48]. [49] found that when accounting for congestion in a classic urban form model, optimal density is much higher than market density, which neglects the social cost of congestion.

A very simple model of congestion in the monocentric model is discussed by [50] and employed here. Congestion at radius  $r$  depends on the population living outside  $r$ , denoted by  $\Omega(r)$  (equation

200 (6)).

$$C_{co} = b_{co} \int_0^{r_c} \left( \frac{\Omega(r)}{2\pi r} \right)^m dr \quad (7)$$

$b_{co}$  is a scaling factor,  $\Omega$  is as in equation 6.  $m$  introduces a non-linearity, we use  $m = 2$ . This formal approach to congestion modelling is fairly basic, however, it provides a straightforward description of congestion based on the urban density profile. Note that the model discussed here does not include a feedback of congestion on the urban density profile. We discuss further shortcomings in sections 3.1 and 4.5.

#### 2.2.4. Infrastructure cost and aggregate utility

The relationship between urban form, modal share, and overall utility is complicated and deeply entangled with a number of factors, such as income and marginal transport costs. Urban economic theory suggests a strong relationship between urban form and transport costs during the historical development of a city: higher marginal transport costs cause a denser growth of urban areas. Clearly, however, in urban planning, fuel prices are usually not considered a policy instrument, as it is outside the repertoire and responsibility of urban planners. First, for economic agents this means that a higher share of their income will be invested into transportation costs and land rent, and thus their overall utility decreases if everything else remains equal. This relationship between urban density and transport costs holds for many examples throughout the world [10, 11, 51]. In turn, higher income and lower transport costs translate into urban sprawl and higher utility within the native urban economic framework. Second, the introduction of public transport reduces marginal costs of travel for those with access to it, which increases utility. Third, the public transport infrastructure needs to be paid for, with especially high costs for low-density developments: lower density developments support lower public transport ridership numbers and hence reduce the return-on-investments for the infrastructures [27]. High-density public transport infrastructure can be very expensive to build on a per-distance basis, both due to high land-prices and expensive engineering (e.g. subway tunnels). Such investments can usually be justified, however, by high passenger numbers [52, 11]. Wealthy cities tend to have a higher car modal share, particularly compared to developing countries and under comparable transport policies. Wealthier municipalities have more public transport funding available, pointing into a different direction than the urban sprawl effect of higher incomes. Altogether, the availability of a municipal budget appears to be a crucial

intermediate variable. However, some examples, such as the creation of good public transport infrastructure in former Sovjet Republics despite budget constraints, depicts the necessity of political willingness. In so far as the financing of the infrastructure could be recovered by marginal user fees and higher land rents (land value capture), the issue, at least in theory, is less one of municipal budget but of financing institutions. We have chosen to capture these dynamics by assuming that the public transport system is financed by municipal taxpayers. This is captured in a conceptual fashion as described below and outlined in equation 8.

Overall utility effects of dense developments are difficult to evaluate, as higher density and higher accessibility may support additional economics of density. These include, for example, those that rely on face-to-face communication such as banking [53] and other areas where agglomeration economics play a strong role.

We conceptualize possible impacts of public transport infrastructure financing by accounting for it in the population’s utility. We use an approach in which the endogenous utility from the urban economics model is modified by three components. Public transport users are supposed to have their utility increased because savings in transport payments (due to  $m_{pt} < m$ ) gives them a higher bread consumption. However, all inhabitants suffer a utility loss from financing public transport infrastructure, which depends on the size of the city. To reflect this, we introduce an impact called *aggregate utility* ( $u_{agg}$ ), which accounts for these effects. It is the weighted average for the city’s population as specified by:

$$u_{agg} = \eta_{car}u + \eta_{pt}u_{pt} - u_{is} \quad (8)$$

This approach is described in more detail in Appendix B. Time costs, the possibility of increased dwelling space or dynamic feedbacks onto the urban density profile are not accounted for in this study. Possible impacts of dynamic effects are discussed in section 3.1.

### 3. The sustainability window of urban form

The interaction between the urban transport system and population density shapes a distinct sustainability outcome for each of the four environmental and socio-economic concerns (a) GHG Emissions, (b) air pollution, (c) road congestion and (d) aggregate utility. Figure 1 displays the sustainability outcomes as a scatterplots showing a range of cities with different modal shares of

public transport and non-motorised modes and different mean urban population densities. The color-coding qualifies the impact of the respective concerns: green for low, orange for medium and red for high values. The values for urban population density shown in this plot are chosen to fit the selection of cities shown in Figures 4 and 5.

260 The model indicates that urban form and transport parameters lead to varying outcomes for each of the sustainability dimensions. Greenhouse gas (GHG) emissions of urban transport (Fig. 1a) decrease with public transport and non-motorised modal shares, and with population density. Per-capita GHG emissions are lowest in dense cities with a high share of public and non-motorised transport.

265 Air pollution from traffic (Fig. 1b) shows a different dependency. In the model, air pollution decreases with non-motorised transport share, but increases with urban density. High urban density leads to a high intake fraction and therefore cause a high health impact of air pollution, despite lower emissions of pollutants. This means, that the impact of air pollution is lowest in low-density cities with a high share of public transport.

270 For congestion (Fig. 1c), the model suggests a qualitatively similar image to GHG emissions. However, the influence of urban density is not straightforward and, the model results are not confirmed by empirical data: The model predicts higher congestion with low density, while empirical analysis suggest the opposite. This contradiction is discussed, and conceptually resolved, in sections 4.3 and 4.5.

275 The aggregate utility that we compute in our model is depicted in Fig. 1d (see 2.2.4 for a detailed description). Note that, in contrast with the other dimensions, a high utility value is perceived as a positive outcome. The aggregate utility increases with population density and decreases with an increasing public transport modal share, resulting in two areas of high values: Sprawling cities with high car transport shares, and very dense cities with high public transport modal shares.

280 We mask the most detrimental areas in each concern that is shown in the panelplot (Figure 1) on top of each other and the obtain a 'sustainability window' of urban form. This sustainability window is shown in Figure 2. Graphically, the sustainability window is constituted by the area indicated by the green areas. Distinctively, the best overall value in all dimensions seems to be achieved in the areas of medium urban density and high public transport plus non-motorised transport modal shares. Scaling the urban population density from the urban economics model to fit the range found  
285 in an empirical dataset (section 4) we find that cities have the most favorable impact on climate

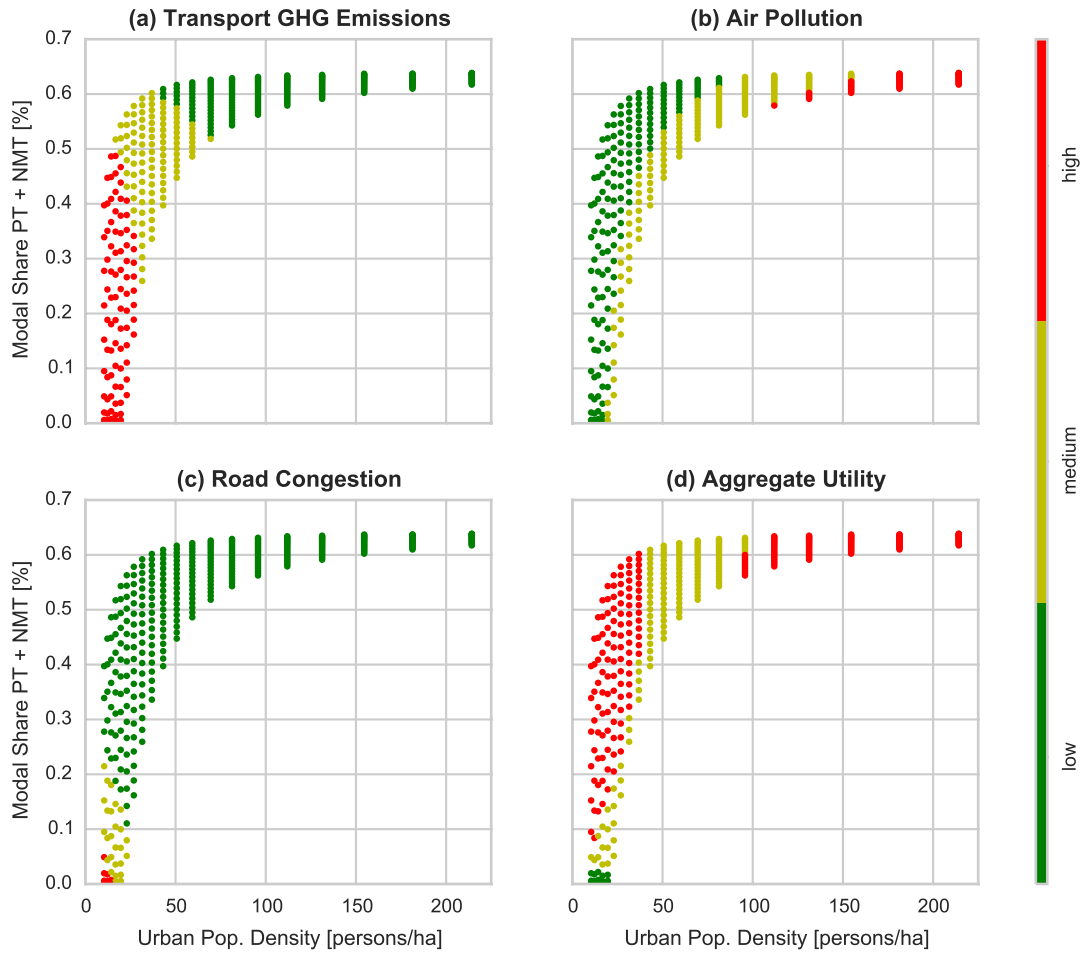


Figure 1: Scatterplots depicting the four environmental, social and economic concerns: (a) GHG Emissions, (b) air pollution, (c) road congestion and (d) aggregate utility for different city types. The colors show the model output for different combinations of urban population density and public transport plus non-motorised transport modal shares. The population density values provided are scaled to fit the selection of cities shown in Figures 4 and 5. The dots are color coded to depict low, medium and high values of the four dimensions normalised to their maximum values. We have chosen not to quantify the numerics more precisely in order to focus the attention on the conceptual idea we are presenting. GHG emissions colors are plotted on a logarithmic color scale. High utility (red) is considered as a positive outcome.

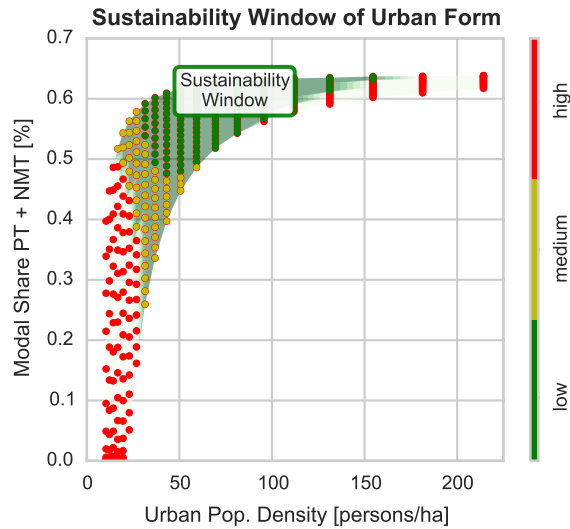


Figure 2: This graphic uses all four panel plots from Fig. 1 and combines them using a weighting function, masking all the points which do not suffice sustainability standards. The dots which are red are supposed to be unreasonable in at least one of the four dimensions. We refer to the green area as the 'window of most sustainable urban form' or 'sustainability window'.

change, air pollution, congestion and urban price index when population density is between 50 and 150 persons/ha and when public transport modal share exceeds 50% (60% for population densities above 100 persons/ha).

290 Clearly, the specific shape and location of the sustainability window depends on the relative weighting of the environmental and socio-economic dimensions. Any specific policy advice could not be based on this model alone, but would require a more detailed approach. However, we can make use of the sustainability window to demonstrate the effect of policy options and technological advances. For example, if vehicles switched from gasoline to electric propulsion, on-street air  
 295 pollution would be massively reduced, making the air pollution impact of urban transport a much smaller issue. As a result, the window of sustainability would be increased for cities with very high population density. This is shown in Figure 3b. In another example, public policies put a higher weight on climate change mitigation. Then, the minimal requirements on both population density and public transit modal shares would increase. The effect on the sustainability window can be  
 300 seen in Figure 3a.

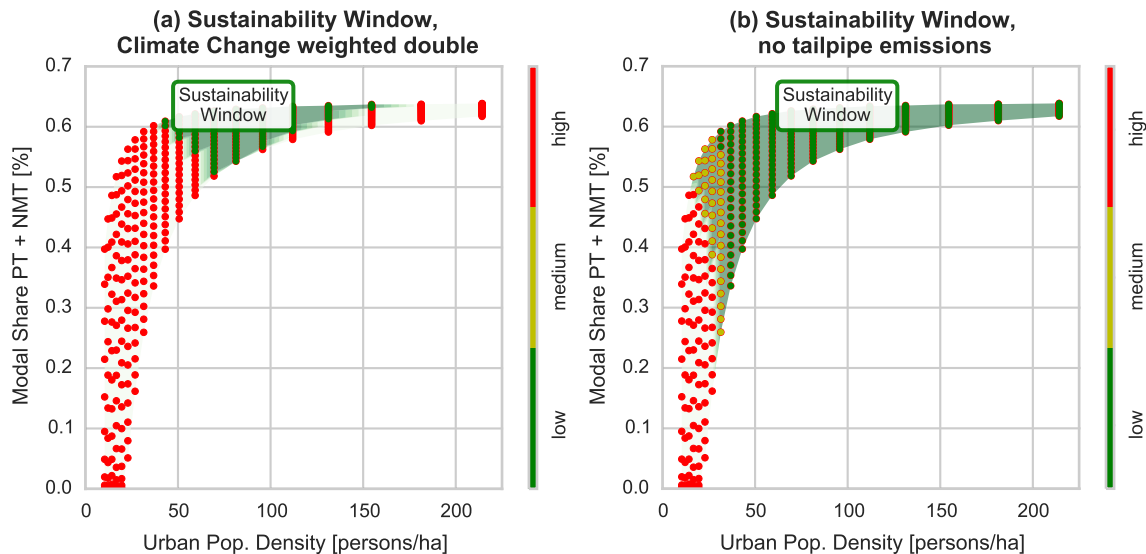


Figure 3: Modifications of Figure 2. (a) shows the impact of GHG emissions from transport is doubled, and (b) shows a the equivalent sustainability window for a case without air pollution impact. The former could correspond to more serious climate change mitigation efforts, the latter could represent the (extreme) case of a city with exclusively electric cars, trams and cycling leading to the absence of tailpipe emissions.

### 3.1. Dynamic effects of sustainability outcomes

How do the environmental and socio-economic effects dynamically interact with urban form? Many studies exist that formally internalise the dimensions – externalities in an economic framework – as an extension to the basic urban economics model. As examples, [54] endogenizes locational choice with respect to transport availability, [36] formally endogenizes air pollution into a location-choice model, and [46] provides a formal description of the effects that congestion has on the urban density profile. Further examples have been given in section 2.2. From a formal economic perspective, our paper is limited, as it does not consider dynamic, endogenous effects of congestion, air pollution and public transport on urban form. We briefly discuss possible effects as follows.

Dynamically, congestion in a mono-centric city leads to flatter urban land rent and density functions, causing excessive urban extension [55]. Market densities would need an upward correction to

mitigate the external effects of congestion [49]. The effects of congestion pricing on urban form have  
315 been extensively investigated upon, but remain unclear. [51] highlights that urban location theory  
as well as empirical analyses all indicate that decreasing costs of travel support urban sprawl and  
employment concentration, while worsening transportation results in more decentralised and denser  
urban form. However, [51] also points out that these effects are not clear and that empirics show  
numerous influences and effects that are prevalent in the land-use transportation system. Air pol-  
320 lution considerations are likely to lead to lower density developments, because those living in high  
density areas are most negatively affected. The dynamic effects of public transport were recently  
analyzed in [11]. A key result was that the dynamic interaction between optimal public transport  
provision and urban form leads to a slightly denser urban core, which, however, has a relatively flat  
density profile due to the lower marginal costs of public transport. Looking at congestion and air  
325 pollution, one of the dynamic effects point to denser cities, and two ones to more sprawling cities.  
The weighting of the different concerns will thus determine the absolute change in urban form due  
to the endogeneity of these sustainability dimensions. However, the sprawling patterns of richer  
economies suggest that, if anything, the congestion and air pollution dynamics effects dominate,  
leading to more sprawling cities, this worsens the climate change costs while alleviating air pollution  
and, to lesser degree, congestion. The individual budget of economic agents is a key variable in  
330 mode choice. When economies become richer, residents tend to switch to car commuting, and thus  
increase the impact of air pollution and congestion. Here, policy makers come into play to mitigate  
these effects, for example by investing into public transport and hence making it more attractive  
as an alternative means of travel.

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Numerous studies investigate the effects of several externalities simultaneously related to urban  
transport, albeit with a largely static urban form. One example are the studies by [21] and [56],  
which analyse the effect of congestion pricing on air pollution, but without feedbacks to urban form.  
A good reason for omitting dynamic feedbacks, though, is the long-term inertia of urban infrastruc-  
340 ture, housing markets and urban form which plays on much larger timescales than economic agents'  
reactions to certain policy measures. On the other hand, studies such as [57] perform very detailed  
studies of possible future evolutions of a specific city. Exciting and recent advances to overcome  
this situation have recently been addressed by [58], who introduce a systematic modelling approach  
across several disciplines related to urban studies. This approach combines socio-economic models



<sup>345</sup> and land-use models with physical impact models with the goal of policy assessment making. However, to our best knowledge, no study has so far dynamically internalized sustainability outcomes of urban road transport into a formal urban economics setting. This is subject of future research.

#### 4. Analysis of empirical data

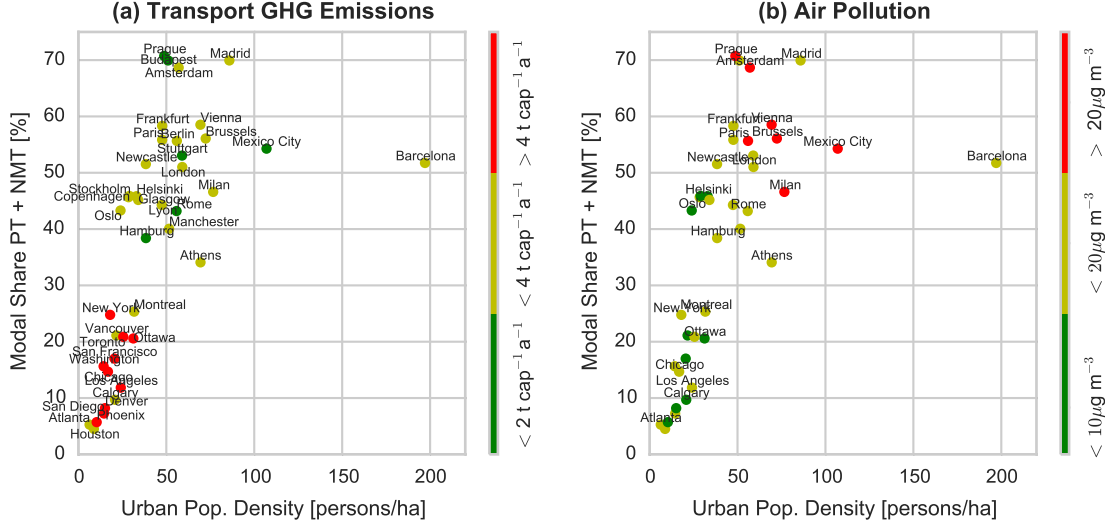


Figure 4: Two scatterplots showing 41 cities plotted in a modal share (public transport plus non-motorised modes) – urban density space and color-coded to denote (a) per capita transport ( $\text{CO}_2$ ) emissions and (b)  $\text{PM}_{2.5}$  concentration levels as a proxy for local air pollution. For  $\text{CO}_2$  emissions: green  $< 2.0 \frac{\text{tCO}_2}{\text{cap year}}$ , yellow  $2.0 - 4.0 \frac{\text{tCO}_2}{\text{cap year}}$ , red  $> 4.0 \frac{\text{tCO}_2}{\text{cap year}}$ . For  $\text{PM}_{2.5}$  concentration: green  $< 10.0 \frac{\mu\text{g PM}_{2.5}}{\text{m}^3}$ , yellow  $10.0 - 20.0 \frac{\mu\text{g PM}_{2.5}}{\text{m}^3}$ , red  $> 20.0 \frac{\mu\text{g PM}_{2.5}}{\text{m}^3}$ . Data: [59] and [60].

This section presents how the sustainability concerns vary across different cities. For this, we perform a basic statistical analysis by looking at how modal share of public and non-motorised transport, urban population density, and urban gross-domestic product (GDP) can explain GHG emissions, local air pollution, congestion levels and general cost-of-living. We base the analysis on the dataset of 100 cities reported by the UITP (International Union of Public Transport) [59], as it focuses explicitly on a correct representation of urban transport statistics. We then added data on per-capita  $\text{CO}_2$  emissions and on air pollution ( $\text{PM}_{2.5}$  concentration) published by the OECD [60], congestion levels from a TomTom traffic survey [61] and a price index as a proxy for the cost of living [62]. Taking into account data availability from all sources, 41 cities remain in the analysis. Given that this relatively small size imposes serious limitations on statistical analyses, we have carried out further analyses taking into account per capita transport energy use and  $\text{NO}_x$  pollution levels from the UITP data set as additional proxies for GHG emissions and air pollution (86 cities in

total here), and investigate the impact of air pollution by analyzing intake fractions (IF) obtained from [63]. We also identified existing literature that substantiate our observations in each section. The statistical models tested explain only the data partially. Data quality might be impacted by inconsistent use of urban spatial boundaries in data collection, and the small sample size.

365 Figs. 4 and 5 contain the 41 cities plotted in a modal share - urban population density space. The large North American cities are located in the bottom left corner with urban densities of around 30 persons/ha, while European cities tend to have both higher urban densities and higher NMT modal shares. This plot visualizes that a certain minimum density is required to acquire higher modal shares of non-car transport. The strong spread also shows that local policy measures for  
370 improving public transport and stimulating bike usage – good examples are Copenhagen (cycling) or Madrid (public transport) – are necessary to achieve high modal shares.

We used ordinary least squares models to explain the external dimensions in the empirical data. Table 1 lists the most relevant results from the analysis. GHG emissions are approximated in emitted CO<sub>2</sub> tonnes per inhabitant  $\frac{t_{CO_2}}{cap\ year}$  [60] and per-capita transport energy use  $\frac{MJ}{cap}$  [59]. Local  
375 air pollution level is measured by the PM<sub>2.5</sub> concentration in units  $\frac{\mu g PM_{2.5}}{m^3}$  [60], by the NO<sub>x</sub> levels per capita  $\frac{kg}{cap}$  [59] and additionally by the intake fraction of pollutants in parts-per-million ppm [63]. Congestion is measured in the "percentage increase of overall travel times when compared to a Free Flow situation" [61]. The price index is an artificial index representing overall living costs in percent, compared to a reference city (Prague) and serves only as a first approximation for  
380 affluence [62]. The gross domestic product (GDP) variable is in GDP per capita for the selected cities in units  $\frac{\$}{year}$ , assuming purchase power parity. The urban population density  $\rho$  is in persons per hectare [59], non-motorised and public transport modal share  $\eta_{pt}$  is the percentage number per trip [59]. We tested several linear regression models using the logarithm of GDP, a linear modal share and the logarithm of population density. We tested both linear and logarithmic dependent  
385 variables, Table 1 shows the model results, which yielded the most convincing outcomes.

#### 4.1. Climate Change

Per-capita CO<sub>2</sub> emissions are highest for low density and low public transit modal shares, but residential population densities above 50 persons/ha seem to suffice to enable low emissions (Fig. 4a). Sprawling American cities such as Los Angeles or Atlanta show particularly high per-capita  
390 CO<sub>2</sub> emissions, a relationship originally investigated by [9] (see [11] for a causal explanation).

Dependent Variable	Data Units	Data Source	$\log(\text{gdp})$	$\eta_{\text{pt}}$	$\log(\rho)$	$R^2$
$\log(\text{CO}_2)$	$\frac{\text{tCO}_2}{\text{cap year}}$	OECD	[0.052]	-0.014	-0.014	0.52
$\log(\text{energy})$	$\frac{\text{MJ}}{\text{cap}}$	UITP	0.27	-0.013	-0.39	0.89
$\log(\text{PM}_{2.5})$	$\frac{\mu\text{g}}{\text{m}^3}$	OECD	[-0.019]	0.0064	0.13	0.30
$\log(\text{NOx})$	$\frac{\text{kg}}{\text{cap}}$	UITP	[0.072]	-0.0071	-0.42	0.41
$\log(\text{IF})$	ppm	Apte2012	-0.12	-0.021	0.21	0.34
congestion	%	TomTom	-2.5	-0.14	6.9	0.29
$\log(\text{price})$	%	Expatistan	0.27	[-0.00088]	[-0.00091]	0.48

Table 1: Regression models of sustainability dimensions, correlation coefficients and p-values from a statistical analysis of a dataset comprising 41 cities (86 for the UITP data, 15 for Apte2012). Coefficients listed are for the following model:  $\beta_0 + \beta_1 \log(\text{GDP}) + \beta_2 \eta_{\text{pt}} + \beta_3 \log(\rho)$ . Brackets around values, e.g.  $[\log(\text{gdp})]$ , denote regression models in which removing the respective variable from the model changed  $R^2$  by less than 0.01 (the results presented always include the variable in question). Data sources: OECD: [60], UITP: [59], Apte2012: [63], TomTom [61], Expatistan [62]. The explanatory variables are all from the UITP source [59].

Here, we proceed by analyzing the relationship between urban form, modal share, GDP and  $\text{CO}_2$  emissions. We have carried out a multivariate regression analysis (Table 1). It reveals both for per capita transport  $\text{CO}_2$  emissions from the OECD data and per capita transport energy use from the UITP data (as a proxy for  $\text{CO}_2$  emissions) that  $\text{CO}_2$  emissions in transport decrease with higher modal share of environmental modes and with higher population density, but increase with affluence. This is exactly in agreement with our theoretical prediction. In the OECD dataset, removing GDP from the regression model changes  $R^2$  by less than 0.01

The question of causality is, however, tantalizing. Urban economics suggests a strong causality between urban density and modal split: higher density enables financially viable public transit and short distances for non-motorized transport [11]. We therefore expect urban density to be the true causal driver of per-capita  $\text{CO}_2$  emissions.

#### 4.2. Local air pollution

Figure 4b suggests that air pollution is acceptable for population densities of about 50 persons/ha and below: The cities below this approximate threshold tend to have lower concentrations of particulate matter. In some cities, such as Amsterdam or Prague, not even a high modal share of non-car transport does help to mitigate air pollution, presumably due to the high population densities in the central cities. We analyzed the relationship between the concentration of particulate

matter (PM<sub>2.5</sub>) and nitrous oxide NO<sub>x</sub> (see 2.2.2) as a function of residential population density, the public plus non-motorised modal share and GDP (table 1). We find inconsistent effects for  
410 all components of the regression model. Taking GDP out of the model changed R<sup>2</sup> by less than 0.01. The air pollution measurements used here reflect concentrations of pollutants. However, the relevant measure, representing the impact of air pollution, as we have modeled, is intake fraction (IF). We therefore consulted a set of 20 cities, reported by [63]. We find that population density alone strongly influences intake fraction ( $R^2=0.61$ ). A regression analysis with a sample of 15 cities  
415 suggests that GDP and modal share of environmental modes influence intake fraction negatively, but that increasing population density leads to higher intake fractions (table 1). This analysis of air pollution outcomes, albeit building on a small data set, is in accordance with our theoretical analysis.

### 4.3. Congestion

420 The congestion data indicates that the situation is more complex than suggested by our model. In fact, some of the highly sprawling cities demonstrate low congestion levels; congestion particularly increases with higher urban density and low modal share of public transit (Fig. 5a). The multivariate regression analysis suggests that congestion decreases with modal share of environmental modes, but increases with density (Table 1). Density has a stronger predictive power than  
425 modal share. The empirical results for congestion do not reproduce the model outcome, particularly for very dense cities, for which the model suggests high congestion levels. The model relies on a monocentric city and bottleneck congestion. Most likely polycentric configurations of more sprawled cities, a specification not included in the model, reduces congestion significantly.

### 4.4. Urban cost of living and affluence

430 As a proxy for the urban costs of living presented in Figure 5b, we rely on a price-index measured for cities worldwide [62]. "Expensive" cities such as London, New York or Oslo stand out: Most cities with high urban costs of living are medium-sized cities from OECD countries, which have a high income and it is very likely that economic affluence would explain these statistics. Indeed, GDP plays the dominant role in the multivariate regression model: higher income leads consistently  
435 to higher price indices and modal share and density play only a small role in the analysis (Table 1).

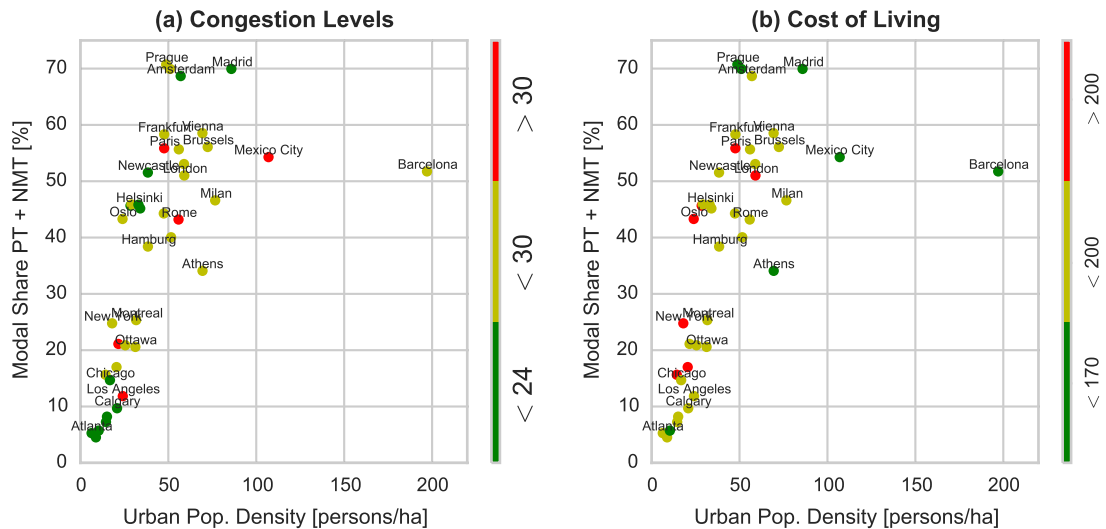


Figure 5: Two scatterplots showing 41 cities plotted in a modal share (public transport plus non-motorised modes)–urban population density space and color-coded to denote (a) relative congestion levels and (b) cost of living. The congestion levels are indicating the overall increase in travel time due to congestion [61]: green < 24%, yellow 24% – 30%, red > 30%. The cost of living index presented by [62] compares urban cost of living to a reference city (Prague): green < 170%, yellow 170% – 200%, red > 200%. Modal share and urban density data from [59].

#### 4.5. Empirical Analysis: Summary

We have performed a multiple regression analysis using the modal share, the urban population density and gross-domestic product as explanatory variables to explain the four different dimensions of urban transport previously discussed: CO<sub>2</sub> emissions, local air pollution, congestion and the overall cost of living. This statistical investigation provides context to our modeling results, the conceptual core of this paper.

The data confirm that CO<sub>2</sub> emissions decrease with higher population density and higher modal share of environmental modes; and that impact of local air pollution (intake fraction) decreases with modal share of environmental modes but increases with population density. The limited size of the dataset, in particular for intake fraction, and the constrained data quality particularly for congestion demand for continued scrutiny in further studies. Other variables, not captured in our statistics, or in the model, may be of equal importance. For example, it is likely that polycentric configurations reduce congestion more than the theoretic prediction in a monocentric model. While the model assumes that lower density leads to higher modal shares of cars that simultaneously

450 enter the city center (see section 2.2.3), sprawling cities tend to also be more polycentric with employment locations and migration trends to suburbia. For example, Houston hosts several business districts, one of them is the Texas Medical Center, which forms a distinct center, remote from the geographical city centre. Polycentricity could hence explain the relatively low congestion costs of low density cities. High-density cities in turn have lower car modal shares, and cars have an altogether 455 smaller street space available. We suspect that an additional measure of policentricity will increase the match between model and data, especially for the congestion metric. Further empirical and theoretical analysis could be the subject of future research, aiming to bridge the gap between idiosyncratic and city-specific factors to general models.

## 5. Discussion and Conclusion

460 This paper investigates how sustainability concerns - or in economic language: environmental externalities - impact what constitutes the optimal urban form and modal share. We show that climate change, air pollution, congestion and cost considerations of citizens point to different and distinct optimal configurations of urban form if considered on their own. This observations prompts us to introduce the 'sustainability window' of urban form. The shape and location of the sustain-  
465 ability window identifies the 'sweet spot', in which urban form and modal share are determined so that all dimensions perform consistently at high quality level. As the main result we find that residential population densities between 50 and 150 persons/ha, and modal shares of public and non-motorized transport combined of more than about 50% are best suited to realize the sustainability window. However, as our paper is mostly conceptual in nature, and data resolution is of  
470 mostly insufficient quality for fine-grained observations, we add that these results are incomplete for specific policy advise of individual cities.

Our results have implications for the call for higher density development coming from those concerned with climate change mitigation. Higher density indeed reduces transport energy use. However, higher density also leads to a utility loss of residents, as they have to live with less  
475 available space. Also, higher density increases the impact of air pollution as more residents are susceptible to inhaling exhaust fumes. Our results highlight that investments into public transport and cycling transport infrastructure can counteract the latter effect by reducing air pollution and congestion in denser inner city areas. Investments into public transport in lower density suburban areas, however, seem less reasonable, as they implicate high financial costs, and less counterfactual  
480 reduction in air pollution.

The empirical analysis presented in section 4, indicated that cities such as Amsterdam, Madrid or Vienna lie in the 'sustainability window'. Crucially, more than 90% of the reduction of GHG emissions - measured against the most sprawling cities such as Los Angeles or Denver - occurs with an increase of population density up to 50 persons/ha. Such a reduction is caused by reduced  
485 commuting distance. At a threshold of 50 persons/ha, modal shift gains additional importance for further reduction of CO<sub>2</sub> emissions. Intermodal concepts such as car-sharing and ride-sharing in combination with public transport infrastructure, and modal shifts enabled by more cycling-friendly streets and less car-focused infrastructure, can realistically only be realized in compact cities. These density thresholds co-align with detailed empirical observations on employment density shown by



490 [64], which identifies a population density of 50 persons/ha as crucial for an increase of public transport modal share, and with a modeling study focusing on the dynamic interaction between urban form and modal share [11]. Interestingly enough, residential non-transport emissions are also significantly reduced and modified above a threshold of 50 persons/ha in one study [28].

The form of the sustainability window changes with technological development. For example, 495 if cars are equipped with highly efficient pollution filters or switch from liquid fuels to electricity, air pollution is drastically reduced (Figure 3b). Then, the optimal modal share tips slightly over towards more cars. More generally, advanced technology favors less dense cities with more cars. In contrast, congestion indicates a hard boundary to urban car transport which can only be overcome by mass-transit and other mobility alternatives to car such as cycling.

500 We briefly discuss this study's caveats. We focus exclusively on residential urban density and modal shares as factors shaping sustainability outcomes. Clearly, however, other factors can be at least equally important. As pointed out above, fuel standards, shifts to electric vehicles or bikes play a crucial role in reducing air pollution and are by no means discussed here exhaustively. Congestion depends on further factors, such as polycentricity, the design of the traffic systems, as well as on parking management or congestion charging policies [43]. In addition, economic affluence 505 shapes all of the sustainability outcomes. A certain municipal budget is required to finance transport infrastructure, while the affluence of citizens also influences their choice of residence and transport mode. Future studies could include and address both empirically and analytically many more of these factors, possibly requiring different modelling approaches and higher resolutions.

510 In summary, our study provides evidence for a 'sufficiency' level of urban population density needed to achieve the sustainability effects studied in this paper. However, higher population density would increase welfare outcomes if air pollution could be technologically controlled. The optimal mix of population density and modal share values cannot easily be obtained through short-term policy measures. A combination of push, pull and land-use instruments, however, especially 515 of vehicle pricing measures together with coordinated urban planning, would enable the majority of cities worldwide to achieve a more sustainable urban form in the long run. Long-term temporal dynamics associated with urban infrastructures alongside issues of path dependence need to be taken into account when studying adequate transition to more sustainable cities. This is a topic we leave open for further studies.

520 **Acknowledgements**

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## Appendix A. AMM model runs

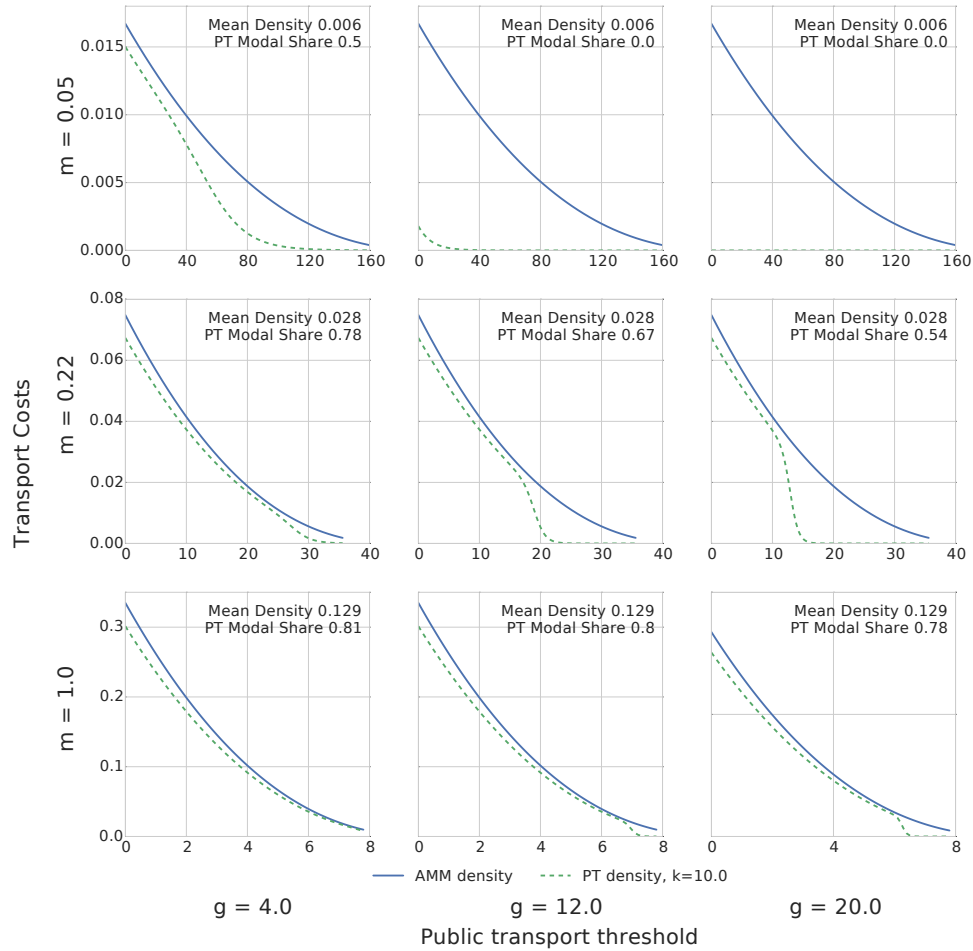


Figure A.6: Density curves of the AMM model run. The solid lines represent the endogenous density profile from the monocentric city model, the dashed lines represent the density profile of public transport / NMT users. The model is run in a finer resolution (in terms of  $m$  and  $g$ ), but figure shows the maximum range of parameters used in the model runs.

525 **Appendix B. Utility**

The infrastructure cost for operating public transport infrastructure is approximated using the product of city size and public transport modal share, corrected with a parameter epsilon:

$$u_{is} = \epsilon_{IS} \cdot A_{city} \cdot \eta_{pt} \quad (\text{B.1})$$

Households using public transport infrastructure are expected to have a higher utility than households using cars, because the marginal cost of transport is lower with public transport infrastructure.

530 These money savings are used by the households to increase their bread consumption. The public transport cost is modeled using a factor  $f_{pt}$  with the condition.

$$0 < f_{pt} < 1 \quad (\text{B.2})$$

This yields the marginal transport price of public transport users:

$$m_{pt} = f_{pt} \cdot m \quad (\text{B.3})$$

The bread consumption of a household is specified using the budget equation

$$z = Y - mr - qs \quad (\text{B.4})$$

and for every household using public transport:

$$z_{pt} = Y - m_{pt}r - qs = Y - mf_{pt}r - qs \quad (\text{B.5})$$

535 Condition (B.2) gives  $z_{pt} > z$ . The difference between the two consumption quantities, i.e. the additional bread consumption of public transport users

$$\Delta z = z_{pt} - z = (1 - f_{pt})mr \quad (\text{B.6})$$

and with  $z_{pt} = z + \Delta z$  these households have a utility

$$u_{pt} = \alpha \log(z_{pt}) + \beta \log(s) \quad (\text{B.7})$$

Typically,  $u_{pt} > u$ . The utility combines the generic utility from the AMM model, the infrastructure costs and the elevated utility of households using public transport. The utility components are

540 weighted with the modal shares of the respective transport modes:

$$u_{combined} = \eta_{car}u + \eta_{pt}u_{pt} - u_{is} \quad (\text{B.8})$$

## Appendix C. Code

The Python routine consists of a main routine, subfunctions and a number of plotting routines for visual output. The formulas described above are coded into the subfunctions. The main routine calls the subfunctions and compute the urban density profile and to calculate the externalities associated with the different density profiles.

The main routine (called `AMM_basic`) is an iteration to find the density profile  $\rho(r)$ . The density curve is given by the standard economic model for a given population number  $N = 1$  and a given transport cost  $m$ , using equation 2 for the bid rent  $\Psi(r)$  and equation 3 for the dwelling space  $s$  at distance  $r$ . This iteration assumes an initial utility  $u = u_0$ , the density profile simply results through  $\rho(r) = \frac{1}{s}$ , and  $u$  needs to be adapted so that the correct population count is achieved: The population count for the density profile is denoted with  $N^*$ , and presumably  $N^* \neq N$ , depending on the initial guess of utility. This utility is re-guessed according to the difference between  $N^*$  and  $N$ , and the iteration stops as soon as  $|\frac{N^* - N}{N}| \leq \epsilon$ .  $\epsilon$  is a small number.

Once the density profile is determined, the equations for the externalities can be applied to it and the externalities can be calculated. AMM iteration:

```
def AMM.basic(r, m, y, u0, alpha, beta, N):
    Nstar = 0
    ustar = u0
    while np.abs(N - Nstar)/N > 0.003:
        rc, rci = city_boundary(r, m, y, Ra)
        Tr = m*r[0:rci]
        psi = bid_rent(y, Tr, ustar, alpha, beta)
        s, rho = dwelling_surface(y, Tr, ustar, alpha, beta)
        Nstar = population_1D(rho[0:rci], r[0:rci])
        z = y - m*r[0:rci] - psi[0:rci] * s[0:rci]
        ustar = ustar * (1 + 0.1 * (Nstar/N - 1))

        z = y - m*r[0:rci] - psi[0:rci] * s[0:rci]
        ucd = alpha * np.log(z[0:rci]) + beta * np.log(s[0:rci])
    if np.abs(ustar-ucd[0])/ustar > 0.005:
```

```

        print "Error in AMM function...Utilities not equal!"

def bid_rent(y, Tr, u, alpha, beta):
    psi = alpha**(alpha / beta) *
           beta * (y - Tr)**(1/beta) * np.exp(-u/beta)
575     return psi

def dwelling_surface(y, Tr, u, alpha, beta):
    s = alpha**(-alpha / beta) *
           (y - Tr)**(-alpha/beta) * np.exp(u/beta)
580     rho = 1./s
    return s, rho

def consumption(r, m, psi, s, y):
585     z = y - m*r - psi*s
    return z

def population_1D(rho, r):
    N = sp.integrate.simps(rho, x = r)
590     return N

```

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