Keeping Pigou on tracks: second-best combinations of carbon pricing and infrastructure provision

Jan Siegmeier

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Abstract

Infrastructure plays a key role for decarbonization: when long-lived public infrastructures (for example roads) complement private goods (cars), they may perpetuate carbon-intensive demand patterns and technologies far into the future. Thus, climate policy must combine 'direct' measures such as a carbon tax with shifts in public investment (in the transport example, away from roads and towards infrastructure for public and non-motorized transport).

However, the composition of infrastructure supply changes slowly, and carbon taxation may be politically constrained. This paper analyzes if one instrument should be strengthened if the other is restricted. It is shown that the opposite may be optimal: Intuitively, if no adequate clean infrastructure is provided, polluting should also be penalized less (and vice versa), unless welfare gains from higher environmental quality are very strong.

More precisely, for non-separable public and private goods entering utility, general second-best conditions are derived and applied to a specific utility function. A constraint on public spending composition leaves the (Pigouvian) tax rule unchanged, while a constraint on taxation implies that the composition of public spending now also addresses the environmental externality. Nevertheless, the second-best *level* of either policy instrument is below its first-best when 'dirty' private goods are sufficiently important in utility.

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1   Introduction

The feasibility and costs of climate change mitigation crucially depend on how fast capital stocks can be adapted to the use of low-carbon technologies. This importantly includes public capital stocks *without* direct greenhouse gas (GHG) emissions: When public goods complement private goods, they may perpetuate GHG-intensive demand patterns and technologies. For example, transport infrastructures and related urban form affect GHG emissions via the number and distance of trips and the choice of transport mode (Sims et al., 2014; Seto et al., 2014). These perpetuating effects are amplified by particularly long lifetimes of infrastructures (Jaccard and Rivers, 2007; Shalizi and Lecocq, 2010). As a consequence, staying within an 'emissions budget' consistent with a $2\,°C$ target may even require the retirement of *existing* infrastructure (Guivarch and Hallegatte, 2011).

Thus, first, infrastructure policies are important mitigation instruments, along with financial incentives or regulations that address GHG emissions directly (Shalizi and Lecocq, 2010) – and both types of instruments need to be adjusted to each other. For example in transport, a fuel tax is inefficient at reducing car use if there are no viable alternatives, so sufficient infrastructures for public and non-motorized transport are required (May and Roberts, 1995). Similarly, subsidies for buying or using electric vehicles are ineffective without a network of publicly accessible charging stations.

Second, if infrastructure cannot be adapted optimally (at least in the short run),[1] the question arises if this should be compensated by a higher carbon price. Vice versa, if the latter is politically restricted, should more public funds and public space be allocated to infrastructure that supports 'clean transport', and less to roads and parking? Clarifying this requires a *specific second-best analysis* of interactions between carbon pricing and infrastructure investment.

However, the role of infrastructures in environmental policy is often neglected both by environmental economics[2] and public economics (see below). In particular, the consequences of politically decisive constraints on environmental taxation and public spending have been left unexplored, although these may be the rule rather than the exception in practical policy-making.

In this paper, I thus analyze the link between two policy instruments: an environmental tax, and the ratio of spending on two public goods which are complementary to clean and dirty private consumption goods, respectively.

The main result is that if one policy instrument is restricted, it is *not* always welfare-enhancing to reinforce the other instrument to compensate

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[1] In addition to long capital stock lifetimes, infrastructure adaptation may be impeded by long planning and construction times, administrative and legislative obstacles, or public opposition.

[2] For example, the Handbook of Environmental Economics (Mäler and Vincent, 2003) does not consider infrastructure investment or urban planning.

this, but it may be necessary to weaken it as well. More precisely, this holds for the case that dirty consumption relative to clean consumption, and composite consumption relative to environmental quality, are 'sufficiently important' for individuals' utility: then, if either the environmental tax rate or the public spending ratio is constrained below its first-best level, the second-best level of the other policy variable is also *lower* than its respective first-best. Further, a tighter constraint means an even lower second-best level of the other policy variable.

Intuitively, to the degree that alternative infrastructure cannot be provided to its optimal level (for example for public transport), polluting behavior should also be penalized less (for example by a fuel tax), unless welfare gains from higher environmental quality are very strong (and similarly for constrained taxation).

The results are formally derived in a static model, but also have an important interpretation in dynamic settings: the speed at which the environmental tax rate should be increased to its first-best is determined by the speed of public capital stock restructuring. Given the often substantial planning lead time and lumpiness of infrastructure investments, they should *precede* environmental tax increases. Such a long-term perspective is not always adopted, as I will illustrate using the example of rail and road infrastructure investments in countries that have committed to climate change mitigation.

A more general implication is that attempts to affect private behavior for example by financial incentives should take into account the infrastructure. In turn, infrastructure can be used to actively shape private behavior, rather than just 'matching' demand. In particular, when taxes or public spending are restricted, integrated policies are required.

The present paper fills a gap in the public economics literature, in which environmental policy is often treated as a topic in optimal taxation: environmental taxes are analyzed as a source of public funds, in settings that are second-best because no lump-sum taxes are available. Models of optimal environmental taxation standardly include public goods – but they are commonly assumed to be (weakly) separable from private consumption, or there is only one public good (Bovenberg and Goulder, 2002), so the effect of the composition of public spending on the environment cannot be elucidated. As an exception, Bovenberg and van der Ploeg (1994) model two public goods and allow for public-private complementarity. However, they again only use it for the analysis of second-best settings without lump-sum taxes, and to determine the optimal level of public goods that are themselves polluting or emission-reducing.

A few numerical studies on climate change mitigation pathways examine the link between carbon pricing and infrastructure: Waisman et al. (2012, 2013) illustrate that a given abatement target can be achieved by a lower

carbon price (and at lower costs) when it is combined with transport-specific policies, including a given recomposition of transport infrastructure investment. In an urban context, Avner et al. (2014) show how public transport infrastructure increases the price elasticity of CO_2 emissions, and thus the efficiency and effectiveness of a carbon- or gasoline tax. However, these studies are numerical, the composition of infrastructure investments are chosen *ad hoc*, and no attempt is made to find the *optimal* public policy to complement carbon pricing.

This paper proceeds as follows: Section 2.1 describes a model in which environmental quality enters utility, and two types of public spending complement a clean and dirty private good to yield 'green' and 'brown' composite goods, respectively. Section 2.2 shows that the government can induce a shift towards clean consumption either by a change in the environmental tax, or by providing a different combination of public goods. The relative size of the marginal effects depends on the elasticity of substitution between 'green' and 'brown' composite goods, and on the elasticity of the composite goods with respect to their respective public input.

Section 3 derives general optimal policy conditions, and Section 4 applies them to a (CES-based) utility specification, each for first- and second-best cases:

In the first-best benchmark, when a Pigouvian environmental tax internalizes the entire environmental damage, the optimality condition for the accompanying composition of public spending simply requires equal marginal utilities of composite consumption across public goods – independently of environmental quality or any private-public complementarity (Section 3.1). With a specific utility structure, this implies that the composition of public spending equals the composition of private spending (Section 4.1).

In the second-best, a binding constraint on the composition of public spending leaves the Pigouvian *tax rule* intact, but the *tax rate* will be different, because the marginal utility of dirty consumption depends on the composition of public spending (Section 3.2.1). If the parameters in the exemplary utility function are such that dirty consumption is sufficiently important, I find that the second-best tax is lower than in the first-best, and that the more the composition of public spending is restricted, the lower the tax (Section 4.2.1).

If there is a binding constraint on the environmental tax rate, the second-best condition for public spending also contains the marginal effects of public spending on the environment (via dirty private consumption). This reflects the role of the composition of public spending as an environmental policy when the environmental tax does not fully internalize damages (Section 3.2.2). However, if dirty consumption is again specified to be sufficiently important, the second-best share of 'clean public spending' is lower than in the first-best, and the more the environmental tax is restricted, the lower

the share (Section 4.2.2).

Section 5.1 discusses implications in a dynamic setting, and demonstrates that transport infrastructure investments in many countries do not yet match long-term climate change mitigation objectives. Section 5.2 discusses variations of the model.

Section 6 concludes.

2 A model of private consumption with complementary public goods

This section first describes a model of household consumption for analyzing the role of public spending composition as an explicit policy instrument (in combination with environmental taxation). The key assumption is a utility function in which clean and dirty private goods are each complemented by a specific public good (rather than being separable).[3] Then, the households' optimality conditions are used to derive the private response to marginal changes in arbitrary government policies. Later sections will consider optimal policies.

2.1 Production and household consumption

Production in our model is very stylized. There are N identical households each supplying one unit of labor as the only input to production.[4] Output can be used for clean or dirty private goods (C, D) or two corresponding public goods (X, Z). Thus, the commodity market equilibrium is[5]

$$N = NC + ND + X + Z. \tag{1}$$

The representative household's utility is

$$U = U\left\{Q[G(C, X), B(D, Z)], E\right\}. \tag{2}$$

Here, $E = E(ND)$ denotes environmental quality (with $E_{ND} < 0$), which is assumed to be (weakly) separable from all other inputs to utility. In contrast to the standard model, private consumption and public spending are not separable: private goods C, D combined with public goods X, Z yield 'green' and 'brown' composite intermediate goods G, B, respectively,

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[3]A similar model could be constructed for the production side, with clean and dirty private inputs complemented by two types of public goods and environmental quality as a (weakly) separable input.

[4]To focus on the relation between environmental taxation and the composition of public spending, I neglect the private choice between labor and leisure and allow for lump-sum taxation to balance the government's budget.

[5]Units have been normalized such that the constant rates of transformation is set to one.

which in turn determine the subutility of consumption Q. Furthermore, assume that $G_i > 0$, $G_{ii} \leq 0$, $B_k > 0$, $B_{kk} \leq 0$, and that C, X and D, Z are complements in the sense that $G_{ij} > 0$ and $B_{kl} > 0$ (with $i, j \in \{C, X\}$, $k, l \in \{D, Z\}$ and $G_i := \partial G/\partial i$, etc.).

Households take prices, public spending and environmental quality as given and maximize (2) subject to their budget constraint

$$C + (1 + \tau)D = 1 - T, \tag{3}$$

where $\tau \geq 0$ is a tax on dirty consumption and T a lump-sum tax. The first-order optimality conditions are

$$U_C = \lambda, \tag{4a}$$
$$U_D = \lambda(1 + \tau), \tag{4b}$$

with λ denoting the marginal utility of income. Thus, the marginal rate of substitution (MRUS) between dirty and clean private goods is

$$\frac{Q_D}{Q_C} = \frac{Q_B B_D}{Q_G G_C} = 1 + \tau. \tag{5}$$

If functions G and B are given, the optimal recomposition of private consumption in response to marginal changes in policy parameters can be derived from this condition under the assumption that Q is homothetic (see Section 2.2).

If additionally, functions U and Q are fully specified, we can use (5) and (3) to obtain demands as explicit functions of policy parameters

$$C = C(\tau, T, X, Z), \tag{6a}$$
$$D = D(\tau, T, X, Z). \tag{6b}$$

Section 4 discusses an example based on a constant elasticity of substitution (CES) specification for Q.

2.2   Households' response to marginal policy variations

If Q is homothetic, the marginal rate of substitution between composites Q_B/Q_G is a function of the ratio of composite goods G/B. Totally differentiating (5) then yields

$$\frac{1}{\sigma}(\tilde{G} - \tilde{B}) - (\tilde{G}_C - \tilde{B}_D) = \tilde{\tau}, \tag{7}$$

where a tilde denotes relative changes (except $\tilde{\tau} := d\tau/(1 + \tau)$) and σ is the elasticity of substitution between green and brown composite goods (with $\sigma > 0$). Thus, for a given change in the environmental tax (and public goods

X, Z, on which G, B depend), the optimal adjustment in private consumption balances two effects: the change in the ratio of composite goods, and thus in the marginal rate of substitution between composites, which is the first term on the left-hand side (LHS) of (7); and the change in the ratio of the marginal (sub)utilities of private goods. For general G and B, the LHS of (7) can be expressed in terms of changes in private and public goods, weighted by expressions containing the elasticities of G, B, G_C and B_D (see (A1) in Appendix A.1).

To be more specific, we secondly assume that

$$G(C, X) = C^\alpha X^\delta, \tag{8a}$$

$$B(D, Z) = D^\alpha Z^\delta \quad \text{with} \quad 0 < \alpha, \delta \le 1. \tag{8b}$$

Imposing the same elasticities of G and B with respect to private and public inputs[6] allows us to write (7) in terms of changes in $\Delta := C/D$ and $\Omega := X/Z$, respectively, because we then have

$$G/B = \Delta^\alpha \Omega^\delta, \tag{9a}$$

$$G_C/B_D = \Delta^{\alpha-1} \Omega^\delta, \tag{9b}$$

and thus:

$$\frac{1}{\sigma} \left[\alpha \tilde{\Delta} + \delta \tilde{\Omega} \right] - \left[-(1-\alpha)\tilde{\Delta} + \delta \tilde{\Omega} \right] = \tilde{\tau}. \tag{10}$$

Solving this equation for $\tilde{\Delta}$, we have shown

Proposition 1 (Private response to marginal policy variations)**.** *The composition of private consumption responds to marginal changes in the environmental tax and the composition of public spending according to*

$$[\alpha + \sigma(1-\alpha)]\tilde{\Delta} = \sigma\tilde{\tau} + \delta(\sigma - 1)\tilde{\Omega}. \tag{11}$$

Thus, the government can achieve a change in the composition of private consumption $\tilde{\Delta}$ by arbitrary combinations of changes in the Pigouvian tax $\tilde{\tau}$, to make the dirty good more or less costly relative to the clean good, and in public spending composition $\tilde{\Omega}$ (with a proportionality factor of $\delta(\sigma-1)/\sigma$), to make the dirty private good more or less 'useful' relative to the clean private good.

The term in the square brackets on the LHS of (11) is always positive: an increase in the ratio of clean and dirty private goods Δ positively affects the ratio of green to brown composites (9a), and thus the marginal rate of substitution between composites (the first term on the LHS of (10) or (7)); it also lowers the ratio of the marginal utilities of private goods (9b), corresponding to the second term on the LHS of (10) or (7). Although they

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[6]For the more general case with different elasticities, see (A2) and (A3) in Appendix A.1.

are of different size, both effects work together to increase the MRUS in (5). Such an increase is the optimal response to an increase in the price of D due to a higher environmental tax ($\tilde{\tau} > 0$, $\tilde{\Omega} = 0$), which thus always leads to a cleaner composition of private consumption ($\tilde{\Delta} > 0$), as expected in the two-good case.

More interestingly, a shift of public spending towards the public good complementary to clean consumption ($\tilde{\Omega} > 0$, $\tilde{\tau} = 0$) does only lead to a 'greening' of private consumption if G and B are substitutes ($\sigma > 1$), and otherwise to an unchanged (for $\sigma = 1$) or even more emission-intensive consumption bundle (for $\sigma < 1$). The reason is that an increase in Ω affects the marginal rate of private substitution via the same two channels as described above for Δ – but they now have opposite signs (the effect via the ratio of marginal utilities of private goods is negative), and their relative size is governed by σ (cf. (10)). For $\sigma < 1$, the positive effect via the marginal rate of substitution between composites is stronger and the overall effect of $\tilde{\Omega}$ is positive. This implies that Δ then needs to decrease for the optimality condition to hold.

3   General optimality conditions for taxation and public spending composition

The results so far hold for arbitrary values of the policy variables. Based on Section 2.1, we now derive general optimality conditions for the government's choice of taxes and public spending composition. We start with the government's general welfare maximization problem and then derive optimality conditions for the first-best case when all policy variables (including a lump-sum tax) can be freely chosen, and for two second-best cases when either the public spending composition or the environmental tax are restricted. In Section 4, we will impose more structure on utility to gain further insights for each policy case.

We approach the government's optimization problem by using indirect utility, obtained by substituting the demand functions (6) in (2):

$$V = U\left\{Q\left[G(C(\tau, T, X, Z), X), B(D(\tau, T, X, Z), Z)\right], E(ND(\tau, T, X, Z))\right\}. \tag{12}$$

This a function of the policy variables (τ, T, X, Z), which are chosen by the government to maximize social welfare NV, subject to a budget constraint

$$X + Z = NT + N\tau D \tag{13}$$

and potential restrictions on policy instruments:

$$S \leq \bar{S}, \quad S := X + Z \tag{14a}$$

$$\Omega \leq \bar{\Omega}, \quad \Omega := X/Z \tag{14b}$$

$$\tau \leq \bar{\tau}. \tag{14c}$$

The first-order conditions (simplified by using Roy's Identity, $U_Q Q_\tau = -\lambda D$, and $U_Q Q_T = -\lambda$) are

$$(\lambda - \mu)D - \mu(\tau - \tau_p)D_\tau = \nu/N, \tag{15a}$$

$$(\lambda - \mu) - \mu(\tau - \tau_p)D_T = 0, \tag{15b}$$

$$U_Q Q_X + \mu(\tau - \tau_p)D_X = (\mu - \xi - \zeta/Z)/N, \tag{15c}$$

$$U_Q Q_Z + \mu(\tau - \tau_p)D_Z = (\mu - \xi + \zeta X/Z^2)/N, \tag{15d}$$

where $\lambda = U_C$ is the marginal utility of income, μ is the marginal utility loss of raising one unit of public funds, ξ, ζ and ν are the multipliers associated with (14a)-(14c), and

$$\tau_p := \frac{NU_E(-E_{(ND)})}{\mu} \tag{16}$$

denotes the 'Pigouvian' environmental tax rate.

3.1 First-best policies

General conditions for the first-best environmental tax and spending composition follow directly:[7] If taxation is unrestricted, that is (14c) is non-binding and $\nu = 0$, (15a) and (15b) yield $\mu = \lambda$ and the optimal tax rate

$$\tau^* = \tau_p. \tag{17}$$

Furthermore, unrestricted public spending on both public goods implies (14a) and (14b) are non-binding and $\xi = \zeta = 0$. Using (17) in (15c) and (15d), we then find that optimal public spending X^*, Z^* must satisfy

$$Q_X = Q_Z, \tag{18a}$$

$$N\frac{Q_X}{Q_C} = N\frac{Q_Z}{Q_C} = 1. \tag{18b}$$

These are standard result for optimal environmental taxation and public spending when lump-sum taxes are available, so that the marginal cost of public funds μ/λ is unity (cf. Bovenberg and van der Ploeg, 1994, ch. 4): The optimal tax on dirty consumption in (17) simply internalizes the environmental externality. Condition (18a) ensures the optimal composition of public spending by equating the marginal utility of different public goods. Equation (18b) is the Samuelson condition (with the rate of transformation between public and private goods equal to one).

Interestingly, these results are unaffected by assumptions about the separability of public goods in utility – more generally, they are *independent of how different types of public spending enter utility*: First, the environmental

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[7]'First-best' policies are those that reproduce the social planner's solution, see for example Bovenberg and van der Ploeg (1994); Bovenberg and Goulder (2002).

tax rate according to (17) and (16) depends only indirectly on the composition of public spending (via U_E and U_C, which are functions of X and Z). Second, the terms involving D_X and D_Z in (15c) and (15d), representing the environmental effect of public spending due to its effect on private consumption, have disappeared in (18a) and (18b). Thus, it is optimal to use only the tax to internalize environmental damages.

3.2   Second-best policies

The first-best results above are usually compared to second-best cases in which the government has no access to lump-sum taxes: The second-best environmental tax may then have an additional 'Ramsey component' ($\tau - \tau_p > 0$) due to a revenue-raising motive for differentiated commodity taxation (Sandmo, 1975; Bovenberg and Goulder, 2002). If this is the case, the second-best equivalents of (18a) and (18b) likewise contain terms $(\tau - \tau_p)D_X$ and $(\tau - \tau_p)D_Z$, representing the effect of public spending on revenues from commodity taxation (Bovenberg and van der Ploeg, 1994, ch. 4.2).

In many practical cases, however, environmental policies are further restricted since a full recomposition of public capital stocks takes time (see Section 5.1), or the government's taxation powers may be limited. To focus on this, the second-best analysis in the next two subsections maintains the assumption of lump-sum taxes and considers restrictions on the levels of Ω and τ instead: Limiting Ω does not imply a change to the structure of optimal environmental taxation (as a restricted set of taxes would), but it affects the tax rate (Section 3.2.1). Limiting τ does structurally change the condition for optimal public spending, which now depends on $(\tau - \tau_p)$ even in the presence of a lump-sum tax – however, this is now not due to the effect of public spending on tax revenues, but because due to the role of Ω in addressing environmental damages (Section 3.2.2).

3.2.1   Environmental taxation under restricted public spending composition

Assume that the government cannot adjust public spending composition to its first-best value Ω^*, that is (14b) is binding

$$\Omega = \bar{\Omega} < \Omega^* \qquad (14b')$$

and $\zeta \neq 0$ in (15c) and (15d). If the government can still choose any tax rate, (14c) is non-binding and $\nu = 0$ in (15a), implying $\mu = \lambda$ and

$$\tau' = \tau_p = \frac{N U_E(-E_{(ND)})}{\lambda} \qquad (17')$$

as before. Thus, the second-best tax *structure* is identical to the first-best case. However, the *level* of the second-best environmental tax τ' will gener-

ally be different from the first-best case because restriction (14b$'$) on public spending affects U_E and $E_{(ND)}$:

Consider first the case when the total spending constraint (14a) is binding. Then, second-best X' and Z' can be determined from (14b$'$) and

$$X' + Z' = \bar{S}. \tag{14a$'$}$$

In this case, $\xi \neq 0$ in (15c) and (15d), and (18a) and (18b) are replaced by

$$U_Q(Q_Z - Q_X) = \frac{\zeta}{NZ'}(\bar{\Omega} + 1), \tag{18a$'$}$$

$$N\frac{U_Q Q_X}{U_Q Q_C} = N\frac{U_Q Q_Z}{U_Q Q_C} = 1 - \frac{\xi}{\lambda} - \frac{\zeta}{\mu Z'}, \tag{18b$'$}$$

which can be solved for ξ and ζ.

Second, when total public spending is unrestricted, (14a$'$) does not hold – but (18a$'$) and (18b$'$), simplified by $\xi = 0$, still determine the total public spending level (and ζ).

In either case, the lump-sum tax T can be determined as the 'residual' in the public budget (13).

3.2.2 Public spending composition under restricted environmental taxation

Now, assume that the government cannot adjust the environmental tax to its first-best value and environmental damages are not fully internalized by the tax, so (14c) is binding,

$$\tau = \bar{\tau} < \tau^*, \tag{14c$'$}$$

and $\nu \neq 0$ in (15a). Together with (15b), this implies

$$\mu = \frac{\lambda}{1 - (\tau_p - \bar{\tau})D_T} = \frac{\lambda D - \nu/N}{1 - (\tau_p - \bar{\tau})D_\tau}. \tag{19}$$

It is then optimal to use the composition of public spending to address the environmental externality, additionally to the tax: constraint (14b) is non-binding and $\zeta = 0$ in (15c) and (15d), which leads to

$$U_Q(Q_Z - Q_X) = \mu(\tau_p - \bar{\tau})(D_Z - D_X). \tag{18a$''$}$$

In contrast to (18a), the effects of public spending on dirty consumption and thus on the environment now directly enter this condition for second-best public spending.

If the total spending constraint (14a) is binding, $X' + Z' = \bar{S}$ and (18a$''$) determine X' and Z', and $\xi \neq 0$ in (15c) and (15d). Thus, (18b) is replaced by

$$N\frac{U_Q Q_X + (\tau_p - \bar{\tau})D_X}{\mu} = N\frac{U_Q Q_Z + (\tau_p - \bar{\tau})D_Z}{\mu} = 1 - \frac{\xi}{\mu}, \tag{18b$''$}$$

which can be solved for ξ.

If total public spending is unrestricted, (18a″) and (18b″) simplified by $\xi = 0$ determine the total public spending level.

Again, the lump-sum tax T finally follows from the public budget (13) in either case.

4 Results for a specific utility structure

Further insights can be gained for specific functional forms for utility and environmental quality. We first state our assumptions (extending those in Section 2.2) and the corresponding results for household behavior. This will then be applied in two subsections on first- and second-best policies, respectively.

Assume that overall utility U is of Cobb-Douglas form, with elasticities m, n. For the utility of composite consumption Q, use a CES function with share parameter β and elasticity of substitution σ. For composite goods G and B, use the Cobb-Douglas specification from (8). Finally, assume that environmental quality E has a constant elasticity ϕ with respect to aggregate dirty consumption:

$$U(Q, E) = Q^m E^n, \tag{20a}$$

$$Q(C, D, X, Z) = \left[\beta G(C, X)^\rho + (1 - \beta)B(D, Z)^\rho\right]^{1/\rho}, \tag{20b}$$

$$G(C, X) = C^\alpha X^\delta, \quad B(D, Z) = D^\alpha Z^\delta, \tag{20c}$$

$$E(ND) = (ND)^{-\phi} \quad \text{for} \quad D > 0, \tag{20d}$$

$$\text{with} \quad 0 < m, n < 1, \tag{20e}$$

$$\sigma > 1 \quad \text{and} \quad \rho := (\sigma - 1)/\sigma, \tag{20f}$$

$$0 < \alpha, \delta \leq 1 \quad \text{and} \quad \alpha + \delta < \sigma/(\sigma - 1), \tag{20g}$$

$$\phi > 0. \tag{20h}$$

Parameter condition (20f) implies that G and B are substitutes; the implications of the second part of (20g) (which is satisfied for example if G and B exhibit constant returns to scale) will become clear below. All results below are derived under this set of assumptions without further mentioning. Furthermore, only interior solutions with $C, D, X, Z > 0$ will be formally analyzed, since otherwise E in (20d), Δ and Ω are not defined.

We now describe households' behavior for this specification. Evaluating the private optimality condition (5) and solving for $\Delta = C/D$ then yields

$$\Delta(\tau, \Omega) = \left[\frac{\beta}{1 - \beta}(1 + \tau)\Omega^{\delta\rho}\right]^\eta, \tag{21}$$

$$\text{with} \quad \eta := \sigma/(\sigma - \alpha(\sigma - 1)).$$

Equation (21) describes the relation between *any* level (or change) of policy parameters and private consumption composition, while (11) described only *marginal changes*; the intuition remains the same. When private consumption is optimal, η equals the price elasticity of substitution between clean and dirty private consumption[8]. From (20f) follows that $\eta > 1$ and Δ is a convex function of τ. Since we additionally impose (20g), we have $0 < \delta\rho\eta < 1$ and Δ is a concave function of Ω.

Combining (21) and the private budget constraint (3), we obtain the demand functions

$$C(\tau, T, \Omega) = (1 - T)\frac{\Delta(\tau, \Omega)}{\Delta(\tau, \Omega) + 1 + \tau}, \tag{22a}$$

$$D(\tau, T, \Omega) = (1 - T)\frac{1}{\Delta(\tau, \Omega) + 1 + \tau}, \tag{22b}$$

which are independent of total public spending S. Substituting this back into the utility specification (20) and using $X = \Omega S/(1+\Omega)$ and $Z = S/(1+\Omega)$, we may obtain the indirect utility functions $\hat{Q} = \hat{Q}(\tau, T, \Omega, S)$ (see (A7) in Appendix A.2).

4.1 First-best policies for a specific utility function

To evaluate the first-best tax condition (17), write the Pigouvian tax (16) in terms of elasticities ϵ^{ij} of variable i with respect to j,

$$\tau_p = \frac{\epsilon^{UE}(-\epsilon^{E(ND)})}{\epsilon^{UQ}\epsilon^{QG}\epsilon^{GC}}\Delta(\tau, \Omega). \tag{23}$$

The results of private optimization (21), (22) and (A7) yield

$$\epsilon^{QG} = \Delta(\tau, \Omega)/\left[\Delta(\tau, \Omega) + 1 + \tau\right], \tag{24}$$

while all other elasticities are constant as specified in (20), so

$$\tau_p = \tau_p(\tau, \Omega) = \frac{n\phi}{m\alpha}\left[\Delta(\tau, \Omega) + 1 + \tau\right]. \tag{25}$$

In the first-best, (17) implies that τ on the RHS and τ_p on the LHS are equal.

Furthermore, we can evaluate the first-order condition for the composition of public goods (18a) using the expression for indirect utility (A7).

We thus find that the first-best environmental tax and composition of public spending, τ^* and Ω^*, need to satisfy

$$\tau = \frac{n\phi}{m\alpha}\left[\Delta(\tau, \Omega) + 1 + \tau\right], \tag{26a}$$

$$(\Omega)^{1-\delta\rho\eta} = \left[\frac{\beta}{1-\beta}(1+\tau)^{(\eta-1)/\eta}\right]^\eta. \tag{26b}$$

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[8]Similarly, the elasticity of substitution with respect to the composition of public spending is $(d\Delta/\Delta)/(d\Omega/\Omega) = \delta\rho\eta$.

Using (21) in (26b) reveals a simple relation between first-best private and public spending patterns:

Proposition 2 (Equality of private- and first-best public spending compositions)**.** *The first-best public spending composition equals the private spending composition:*

$$\Omega^* = \frac{\Delta(\tau^*, \Omega^*)}{1 + \tau^*}. \tag{27}$$

Furthermore, substituting (26b) into (26a) yields an implicit (necessary) condition for the first-best environmental tax τ^*:

$$\tau = \frac{n\phi}{m\alpha} \left[\Delta^*(\tau) + 1 + \tau \right], \tag{28}$$

$$\text{with} \quad \Delta^*(\tau) := \left[\frac{\beta}{1-\beta} (1+\tau)^{1-\delta\rho} \right]^{\eta/(1-\delta\rho\eta)}.$$

Here, Δ^* is the ratio of clean to dirty private goods as a function of the first-best environmental tax only. Once τ^* has been determined, the corresponding optimal public spending composition Ω^* can be otained from (26b). Interpreting the left- and right-hand side (LHS and RHS) of (28) as functions of τ permits a qualitative analysis of this condition (see Figure 1):

Assumptions (20f) and (20g) imply that the overall exponent of the $(1+\tau)$-term in the expression for Δ^* exceeds unity, so both Δ^* and the entire RHS of (28) increase at a growing rate in τ, from a positive value for $\tau = 0$. The term in square brackets on the RHS is always larger than τ on the LHS. Thus, a *necessary* condition for the existence of a solution is that utility is less sensitive to changes in aggregate pollution than to changes in composite consumption,[9] that is, if

$$m\alpha > n\phi. \tag{29}$$

To derive a sufficient condition, note that the existence of a solution to (28) requires that the Pigouvian tax on the RHS does not grow 'too fast' in τ, so that the LHS of (28) is tangent to or intersects the RHS (Fig. 1 illustrates the latter case). To be more precise, define

$$M := \frac{m\alpha}{n\phi}, \quad H := \frac{\eta - \delta\rho\eta}{1 - \delta\rho\eta}, \quad R := \left[\frac{\beta}{1-\beta} \right]^{\eta/(1-\delta\rho\eta)}. \tag{30}$$

Then, we can show

Proposition 3. *The first-order optimality condition* (28) *for the first-best environmental tax τ^* has*

$$\left. \begin{array}{l} \textit{two solutions} \\ \textit{one solution} \\ \textit{no solutions} \end{array} \right\} \quad \textit{if} \quad \frac{(M-1)^H}{M^{H-1}} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \frac{H^H}{(H-1)^{H-1}} R. \tag{31}$$

---

[9]Here, the sensitivity to changes in private consumption is measured by $\epsilon^{UQ}\epsilon^{GC} = m\alpha$; since ϵ^{QG} depends on τ and Ω, it does not appear in this condition on parameters. The sensitivity to changes in aggregate pollution is simply the elasticity $\epsilon^{U(ND)} = -n\phi$.
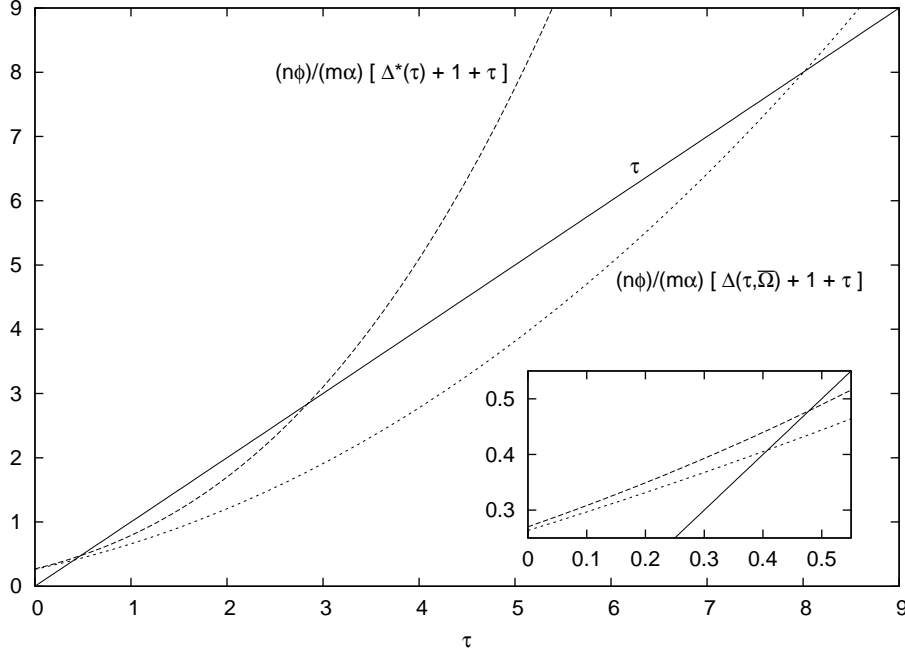
Figure 1: Illustration of the implicit conditions (28) and (32) for the first- and second-best environmental tax. (Parameters: $m = 0.75, n = 0.25, \sigma = 3, \beta = 0.4, \alpha = 0.8, \delta = 0.2, \phi = 0.5, \bar{\Omega} = 0.2$.)

If there are two solutions to (28)*, the smaller one is the solution to the government's welfare maximization problem.*

Proof. The derivation of (31) is technical and moved to Appendix A.3. Identifying the maximum in the two-solution case is more instructive:

The parameter condition (31) implies that there may be two solutions to (28), denoted by τ_L and τ_H with $\tau_L < \tau_H$, if the dirty relative to the clean composite good and overall consumption relative to environment quality are sufficiently important in social welfare.[10] Additionally, $\tau \to \infty$ by (22) implies ever-decreasing D and ever-increasing E by (20d), so that social welfare goes to infinity by (20a). At the other extreme, an arbitrarily small environmental tax will always yield higher social welfare than $\tau = 0$. Thus, τ_L must *locally* maximize social welfare, τ_H locally minimizes it, and there is only *one finite* solution to the government's unrestricted maximization problem: $\tau^* = \tau_L$, and a low $\Omega^*(\tau_L)$ according to (26b). □

Thus, a low environmental tax rate is preferable to an 'intermediate'

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[10]Both the LHS and RHS of (31) increase with σ and α; but if m is large or n and ϕ are small, only the LHS is large, and if β is small, only the RHS is small.
The necessary condition (29) is contained in (31), since the RHS of (31) is always positive but the LHS is non-positive if $M \leq 1$.

tax if dirty consumption contributes relatively strongly to utility. This is only compensated by environmental quality gains if the environmental tax is beyond an upper threshold, which we may denote by τ_{crit}.

That the tax can never be high enough once it is above τ_{crit} is an artifact of our modeling choice for environmental quality (20d). Alternatively, we could for example introduce an upper bound on E (or a lower bound on D), which would be reached at a finite tax rate $\bar{\tau}$: then, if $\bar{\tau} > \tau_{crit}$, this corner solution replaces the interior solution τ_L as the global optimum.

If the environment or the clean composite good are sufficiently important in utility, (31) implies that (28) has no solution. In this case, a potential loss of utility from composite consumption due to a tax increase is always overcompensated by the corresponding increase in welfare due to higher environmental quality: the local maximum and minimum disappear, an increase in τ always increases welfare, and there is *no finite* solution to the first-best problem. Again, a different specification of (20d) would take care of this.

Figure 3 plots the numerical solution to (28) and (26b) for two elasticities of substitution.

4.2 Second-best policies for a specific utility function

We now apply the general results of Section 3.2 and analyze scenarios with binding constraints on policy variables.

4.2.1 Environmental taxation under restricted public spending composition

Using (20)-(22) and (A7) in (17′) gives an implicit condition that a second-best environmental tax τ' needs to satisfy:

$$\tau = \frac{n\phi}{m\alpha}\left[\Delta(\tau,\bar{\Omega}) + 1 + \tau\right]. \tag{32}$$

This equation has the same form as (26a) for the first-best case, only that the exogenous parameter $\bar{\Omega}$ replaces Ω^*, so that (32) can be directly compared to (28) (cf. Fig. 1). In analogy to the first-best case, the following sufficient condition for the existence of solutions can be derived:

Proposition 4. *If there is a binding constraint $\bar{\Omega}$ on the composition of public spending, the first-order optimality condition* (32) *for the second-best environmental tax τ' has*

$$\left.\begin{array}{l} \textit{two solutions} \\ \textit{one solution} \\ \textit{no solutions} \end{array}\right\} \quad \textit{if} \quad \frac{(M-1)^\eta}{M^{\eta-1}} \left\{\begin{array}{l} > \\ = \\ < \end{array}\right\} \frac{\eta^\eta}{(\eta-1)^{\eta-1}} R^{1-\delta\rho\eta}\bar{\Omega}^{\delta\rho\eta}. \tag{33}$$

If there are two solutions, the smaller one satisfies

$$\Delta_\tau(\tau', \bar{\Omega}) < \frac{m\alpha}{n\phi} - 1. \tag{34}$$

If there is only one solution, this holds with equality.

Proof. See Appendix A.3. □

The upper bound $\bar{\Omega}$ may be binding either because it is below the first-best solution for public spending composition, or because there is no finite first-best solution (see above).

Again, (33) implies that $m\alpha > n\phi$ (cf. (29)) is a necessary condition for the existence of a second-best τ'. Otherwise, if environmental quality is too important for utility, there is no interior solution (unlike Fig. 1, which illustrates the two-solution case, the curve representing the Pigouvian tax term on the RHS of (32) is then 'too high and too steep').

If there are two solutions to (32), by the same reasoning as for the first-best case we identify the smaller value of the tax as the one that locally maximizes social welfare, so it represents the unique (finite) solution to the government's second-best optimization problem.[11] Thus, (34) implies that at the second-best, a change in the private composition of consumption in response to a tax change must not exceed a threshold which again depends on the scaling factor of the Pigouvian tax term on the RHS of (32).

Further, we can show

Proposition 5 (Characteristics of second-best environmental taxation)**.** *If a first-best solution (τ^*, Ω^*) exists, there is a second-best environmental tax τ' for any $\bar{\Omega} < \Omega^*$ with*

$$\tau' < \tau^*. \tag{35}$$

Independent of a first-best solution, if $\bar{\Omega}_1$ permits a solution $\tau'(\bar{\Omega}_1)$, under a tighter constraint $\bar{\Omega}_2 < \bar{\Omega}_1$ there is a solution $\tau'(\bar{\Omega}_2)$ with

$$\tau'(\bar{\Omega}_2) < \tau'(\bar{\Omega}_1). \tag{36}$$

Proof. See Appendix A.3. □

Thus, the more public spending composition is constrained, the more the second-best environmental tax will be below the first-best. Both policy variables affect the private consumption composition in the same direction.

In other words, if clean private consumption contributes too little to utility because there is a lack of matching infrastructure to make it useful, dirty consumption (that is relatively well-supported by infrastructure) should not be penalized as much.

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[11]As for the first-best case, a finite corner solution may result from a limit on E or D.

This is illustrated in Figure 1, where the curve representing the Pigouvian tax term on the RHS in (32) is lower than for the first-best case, so its first intersection with the LHS is at a lower τ value. A lower $\bar{\Omega}$ pushes this curve further down.

Figure 3 plots the second-best environmental tax rate (obtained numerically from (32)) as a function of $\bar{\Omega}$, for two different elasticities of substitution.

4.2.2 Public spending composition under restricted environmental taxation

Using (20)-(22) and (A7) first in (19), and then together with this result in (18a″) gives the following implicit equation that the second-best composition of public spending Ω' must satisfy:

$$\Omega = \frac{\Delta}{1 + \tau_p} \left[1 + \frac{\eta}{M} \frac{\tau_p - \bar{\tau}}{\tau_p} (1 + \Omega) \right], \tag{37}$$

with $\Delta = \Delta(\bar{\tau}, \Omega)$ and $\tau_p = \tau_p(\bar{\tau}, \Omega)$. This expression reflects the assumption that $\tau_p \neq \bar{\tau}$ in two ways: The first fraction on the RHS, the ratio of the marginal total (that is, private and social) costs of clean and dirty consumption, does not equal the private spending ratio (see below). The second term on the RHS, in square brackets, implies that if private consumption composition is not optimally adjusted via an environmental tax, the effect of Ω on private consumption composition should be used as a (partial) compensation, instead of simply 'matching' it to private spending (as implied by the first-best condition (27)). More precisely, we can show that the second term is larger than one, so the second-best composition of public spending must exceed the ratio of the marginal total costs of consumption:

Proposition 6 (Characteristics of second-best public spending composition, part I)**.** *If environmental taxation is constrained to $\bar{\tau}$, (37) may have at most two (non-zero) solutions.*
Any solution Ω' satisfies

$$\Omega > \frac{\Delta(\bar{\tau}, \Omega)}{1 + \tau_p(\bar{\tau}, \Omega)}. \tag{38}$$

Proof. See Appendix A.3. □

Again, the upper bound $\bar{\tau}$ may be binding either because it is below the first-best solution for environmental taxation, or because there is no finite first-best solution.

The complexity of (37) severely limits the insights that can be proved analytically. For example, we cannot derive a parameter condition similar to (31) or (33) for the number of its solutions. Instead, Proposition 6 only
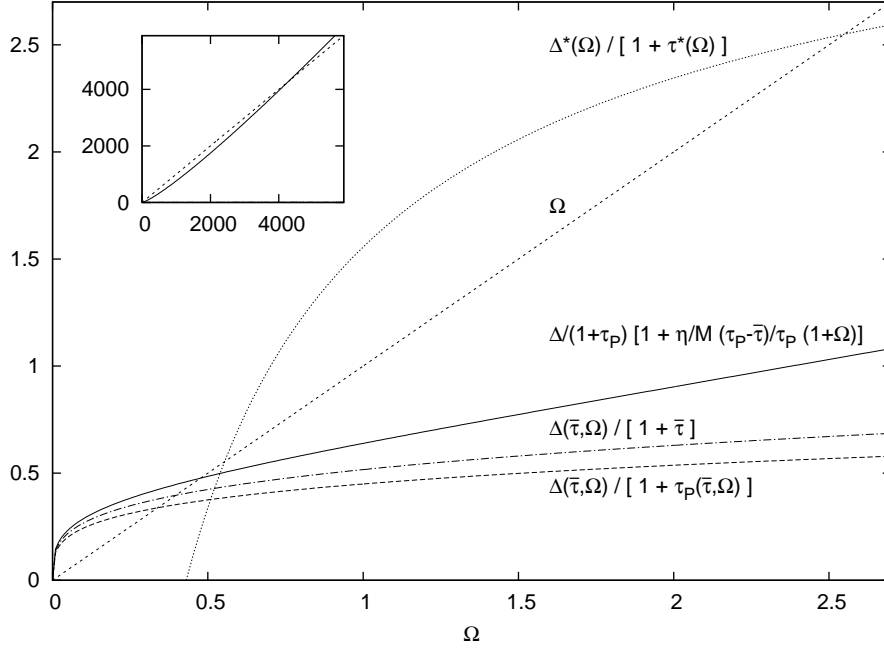
Figure 2: Illustration of the implicit conditions (27) and (37) for the first- and second-best composition of public spending. (Parameters: $m = 0.75, n = 0.25, \sigma = 3, \beta = 0.4, \alpha = 0.8, \delta = 0.2, \phi = 0.5, \bar{\tau} = 0.2$.)

makes a qualitative statement based on the analysis of the shape of the LHS and RHS of (37), interpreted as functions of a general Ω. Figure 2 plots these two functions; their intersections solve the equation.

The same figure illustrates (38), because the curve representing the RHS is above the curve for the ratio of the marginal total costs of consumption at Ω'.[12]

If there are two solutions to (37), it is again the smaller value that locally maximizes social welfare and represents the unique (finite) solution to the government's second-best optimization problem: regardless of any limit on the tax, $\Omega \to \infty$ implies that E goes to infinity, too, and U with it. Thus, the larger value of Ω' must yield a (local or global) minimum of social welfare, and the smaller value a local maximum.[13]

Beyond this, in numerical simulations, I could not find a parameter combination that *qualitatively* changes the relative positions of the intersections of the curves in Figure 2. This leads to

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[12]For the parameters used in Figure 2, this is even the case for *all* Ω. For other parameter combinations, in particular for $\bar{\tau}$ close to τ^*, this may not be the case for $\Omega < \Omega'$.

[13]As above, a finite corner solution may result from introducing a limit on E or D, and may replace the interior solution as the global optimum if it yields higher social welfare.

Conjecture 1 (Characteristics of second-best public spending composition, part II). *Assume that environmental taxation is constrained to $\bar{\tau}$.*

Then, first, the second-best public spending composition exceeds the private spending composition:

$$\Omega' > \frac{\Delta(\bar{\tau}, \Omega'_L)}{1 + \bar{\tau}}. \tag{39}$$

Second, if a first-best solution (τ^*, Ω^*) exists, there is a second-best compositions of public spending Ω' for any $\bar{\tau} < \tau^*$, with

$$\Omega' < \Omega^*. \tag{40}$$

Third, independent of a first-best solutipm, if $\bar{\tau}_1$ permits a second-best solution $\Omega'(\bar{\tau}_1)$, under a tighter constraint $\bar{\tau}_2 < \bar{\tau}_1$ there is a second-best $\Omega'(\bar{\tau}_2)$ with

$$\Omega'(\bar{\tau}_2) < \Omega'(\bar{\tau}_1). \tag{41}$$

Condition (39) in the first part of the conjecture is stronger than (38). In contrast to the first-best condition (27), the private spending ratio is lower than the second-best public spending composition because a constrained environmental tax implies that the former is too small, and that the latter plays a role in changing private behavior.

The interpretation of the other two parts of the conjecture is similar to their counterparts in Proposition 4: The more environmental taxation is constrained, the more the second-best composition of public spending will be below the first-best composition. Both policy variables affect the composition of private consumption in the same direction.

Furthermore, Figure 2 illustrates that even though the *solution* Ω' to (37) converges to Ω^* as $\bar{\tau} \to \tau^*$, the RHS of (37) does not converge to the RHS of (27) *for all other* Ω.

Figure 3 shows numerically obtained solutions of (37) for the second-best public spending compositions Ω' as a function of $\bar{\tau}$, for two different elasticities of substitution.

5 Remarks and extensions

5.1 Dynamic effects and the example of rail infrastructure investment

We motivated our analysis in the introduction with examples from the transport sector, interpreting public spending as investment in different types of infrastructure. Adjusting the composition of these capital stocks typically takes a long time (Shalizi and Lecocq, 2010): planning and construction of new infrastructure takes many years, and existing infrastructure persists for
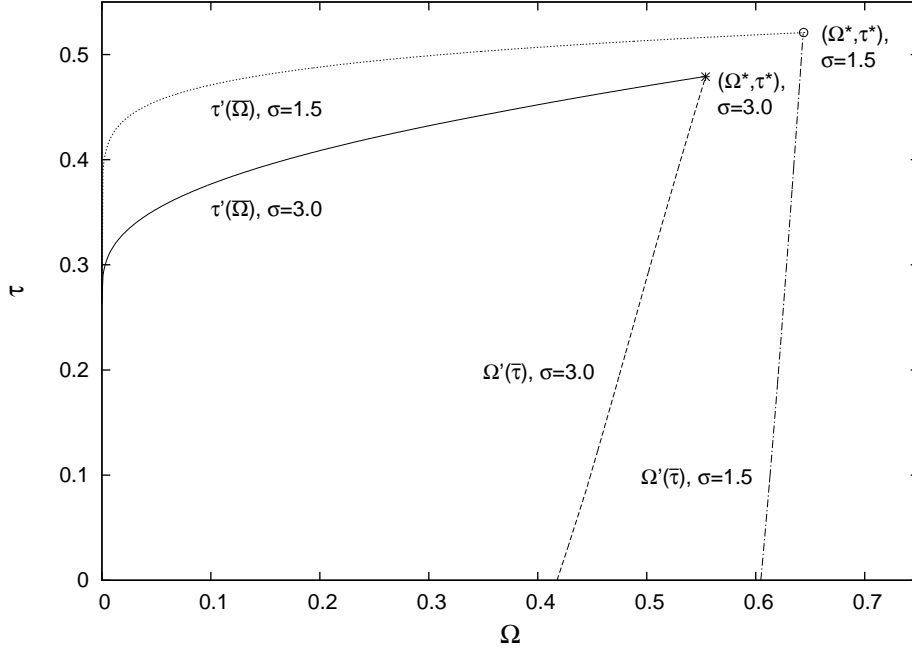
Figure 3: Environmental tax and public spending composition in the first-best case (τ^*, Ω^*) and second-best cases $(\tau'(\bar{\Omega})$ and $\Omega'(\bar{\tau}))$, each for $\sigma = 1.5$ and $\sigma = 3$ (other parameters: $m = 0.75, n = 0.25, \beta = 0.4, \alpha = 0.8, \delta = 0.2, \phi = 0.5$).

decades unless maintenance is stopped altogether or it is actively dismantled, which is often politically difficult. Thus, a first-best analysis with a *static* model as in Section 4.1 only describes a very long-run optimum.

However, the short-run is approximated well by the second-best analysis in Section 4.2.1, in which an environmental tax can be immediately adjusted while the composition of public capital is restricted. Then, it was shown that the environmental tax rate may also have to be lower than the first-best. Thus, in a dynamic setting, the speed at which infrastructure stocks can be restructured to lift the restriction also determines the speed at which the environmental tax rate should be increased towards its first-best. In practice, since infrastructure projects often have substantial planning lead times, they should thus be initiated *before* environmental tax increases. Since a larger supply of clean infrastructure increases the (price) elasticities of demand for clean and dirty goods, this would also facilitate subsequent tax increases politically.

However, such a long-term perspective is not always adopted: For example, Figure 4 plots annual investment in rail infrastructure as a share of total (landbound) transport infrastructure investment in countries that
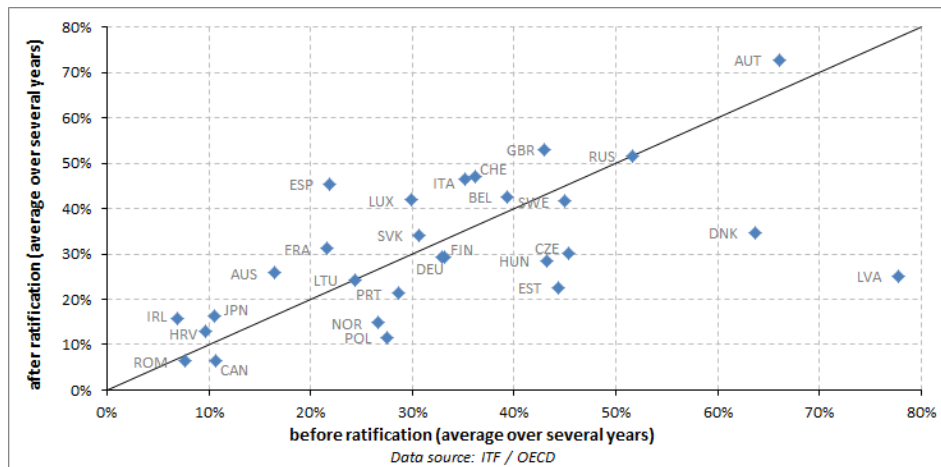
Figure 4: Share of rail in total annual transport infrastructure investment before and after ratification of the Kyoto Protocol

have committed to greenhouse gas (GHG) emission reductions under the Kyoto Protocol, before and after ratification of the protocol.[14] Out of 28 countries, 15 *reduced* their relative spending on rail infrastructure, which is unlikely to be optimal when in the long-run, substantial GHG emission reductions will have to come from the transport sector (Sims et al., 2014). Three more countries increased relative spending by less than 5%, and 19 remained below 40% after ratification.

5.2 Model variants

Public goods in the production function

In our model, environmental quality enters individual utility and public goods affect final demand. But some environmental externalities such as climate change also strongly affect the supply side, and firms' choices between more or less GHG-intensive inputs (such as different transport services) are also affected by public goods and environmental taxes. This would be reflected by a model in which environmental quality and non-separable public goods enter firms' production function. However, for similar functional forms we may expect similar results for second-best taxation and public spending.

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[14]The investment data was averaged over several years before the respective country's ratification, and from the ratification year onwards, as far as data was available from the OECD's ITF (2015).

Different utility function

Our results in Section 4 were derived for a specific utility function. Two assumptions are particularly noteworthy:

First, modeling environmental quality as a function of aggregate dirty consumption by (20d) leads to cases where no finite solution to the government's first- or second-best problem exists. One possible remedy that preserves analytical tractability is to describe environmental quality by (20d) only up to an upper limit, and to keep it constant for lower values of aggregate pollution, as discussed in Section 4.

Second, in (8) or (20c), we imposed the same elasticities of 'brown' and 'green' composite goods with respect to private and public goods (α and δ). A numerical model should drop this assumption, but Appendix A.1 illustrates that the more general case with different elasticities is too complicated for an analytical treatment as in Section 4, and the qualitative insights are unlikely to change.

Network effects

A useful extension for a case-by-case analysis or a numerical model would be to account for network effects in transport infrastructure:

The usefulness of an infrastructure network generally depends on its size, with low marginal benefits of investment for very small or very large networks, and large marginal benefits for intermediate-sized networks. Thus, the elasticities in the model above will not be constant, but functions of the size of the respective network(s), and they will be very different when alternative networks are at different stages of development. This is particularly relevant when considering low-income countries with little infrastructure of any type or economies in transition with major infrastructure investments underway (Shalizi and Lecocq, 2010; Agénor, 2013), where a 'lock-in' on high-emission development pathways may still be avoided.

Labor taxation

To focus on interactions between environmental taxation and public spending, we assumed that lump-sum are available. As mentioned in the introduction, this is in contrast to a large body of public economics literature on environmental taxation in second-best settings in which public funds are costly because they have to be raised via a distortionary (labor) tax (Bovenberg and Goulder, 2002). A central finding of this literature is that the optimal environmental tax rate is below the Pigouvian tax rate, intuitively because it reduces dirty consumption and thus the contribution of the environmental tax to the public budget. More precisely, the optimal tax rate is the Pigouvian tax rate divided by the marginal costs of public funds (MCPF).

We can expect that this still holds when additionally, a constraint on public spending composition is introduced – as in Sections 3.2.1 and 4.2.1, this will only change the *rate*, but not the *structure* of optimal environmental taxation. Vice versa, the other results in 4.2.1 will not change if there is an additional factor (one over the MCPF) in condition (32) for second-best environmental taxation (and all expressions derived from it).

6 Conclusion

This paper analyzed environmental taxation and the composition of public spending when public goods are complementary to private goods. I first demonstrated how either policy instrument can be used to achieve a given change in the composition of private consumption. Then I focused on politically relevant second-best settings in which one of the two instruments is constrained.

In general, if the share of public goods complementary to clean private goods in total public spending cannot be increased to its first-best, this changes the level but not the structure of optimal environmental taxation (which is still Pigouvian). On the contrary, a limit on environmental taxation does structurally change the condition for optimal public spending and thus also its level, because the composition of public spending now plays a role in addressing the environmental externality.

To assess the sign of these changes in second-best policies relative to the first-best, I used a specific utility function. First, I proved that in the first-best, the composition of public spending equals that of private spending. Then, I found that if clean private goods and environmental quality are not too important in utility (in terms of their respective elasticities), a constraint on one policy instrument implies that the level of the other instrument should also be lower than its first-best – and the tighter the constraint, the lower the other policy's second-best level. For the case of second-best taxation, this was formally proved, while the complex second-best condition for public spending composition could only be considered numerically.

These results can most obviously be applied in the transport sector, which is relevant for many environmental concerns such as noise, air quality, climate change, land use and biodiversity (Hensher and Button, 2003): various private transport decisions depend not only on relative prices (that can be influenced by environmental taxes and subsidies), but also on complementary policies such as public spending, or land use management and urban planning.

Examples include, for GHG emission reduction, fossil fuel taxation and a shift of investment from road- towards rail infrastructure; to reduce local (urban) environmental externalities, city tolls complemented by infrastructure investment and land-use management in favor of public and non-motorized

private transport modes; and the provision of charging infrastructure to promote electric vehicles.

In particular, since the results suggest that price instruments should be limited as long as the composition of public goods has not been fully adjusted, and since changing settlement patterns, urban form and infrastructure is time-intensive, the latter should be high on the agenda of climate- and environmental policies.

More generally, the results indicate that externality pricing must take into account the infrastructure, and that vice versa, infrastructure spending should not be viewed as a 'downstream' policy, only reacting to private behavior shaped by seemingly more 'direct' instruments such as taxes. On the contrary, public infrastructure can also be used to actively change private behavior, and in realistic settings with constraints on taxes and public spending, integrated environmental policies are required.

Acknowledgements

A   Appendix

A.1   Generalized private response to marginal policy changes

For general subutility functions $G(X, Z)$ and $B(D, Z)$, the total derivative of (11) in the main part is

$$S_C \tilde{C} - S_D \tilde{D} = \sigma \tilde{\tau} - S_X \tilde{X} - S_Z \tilde{Z}, \tag{A1}$$

with

$$S_C := (\theta_{GC} - \sigma \theta_{GCC}), \quad S_D := (\theta_{BD} - \sigma \theta_{BDD}),$$
$$S_X := (\theta_{GX} - \sigma \theta_{GCX}), \quad S_Z := (\theta_{BZ} - \sigma \theta_{BDZ}),$$
$$\theta_{FJ} := \frac{\partial F / \partial J}{F / J}, \quad \theta_{FIJ} := \frac{\partial F_I / \partial J}{F_I / J},$$
$$\text{with} \quad F \in \{G, B\} \quad \text{and} \quad I, J \in \{C, D, X, Z\}.$$

and, as usual,

$$\frac{1}{\sigma} := \frac{\partial (Q_B / Q_G)}{\partial (G / B)} \frac{G / B}{Q_B / Q_G}.$$

If we choose the specific functions

$$G(C, X) = C^\alpha X^\delta \quad \text{with} \quad 0 < \alpha, \delta \le 1, \tag{A2a}$$
$$B(D, Z) = D^\beta Z^\gamma \quad \text{with} \quad 0 < \beta, \gamma \le 1, \tag{A2b}$$

and evaluate all partial derivatives, (A1) becomes

$$[\alpha - \sigma(\alpha - 1)]\tilde{C} - [\beta - \sigma(\beta - 1)]\tilde{D} = \sigma\tilde{\tau} + (\sigma - 1)(\delta\tilde{X} - \gamma\tilde{Z}). \qquad \text{(A3)}$$

Setting $\alpha = \beta$ simplifies the LHS, and $\delta = \gamma$ the last term on the RHS, so that we obtain (11) in the main text.

A.2   Private behavior for a specific utility specification

For illustration of Section 2 and reference in Section 4.2, this appendix completes the solution of the private optimization problem for the utility specification in (20). Combining (21) with (3) we obtain the demand functions

$$C(\tau, T, X, Z) = \beta^\eta X^{\delta\rho\eta}\frac{1-T}{\pi}, \qquad \text{(A4a)}$$

$$D(\tau, T, X, Z) = \left(\frac{1-\beta}{1+\tau}\right)^\eta Z^{\delta\rho\eta}\frac{1-T}{\pi}, \qquad \text{(A4b)}$$

$$\text{with} \quad \pi := \beta^\eta X^{\delta\rho\eta} + (1-\beta)^\eta Z^{\delta\rho\eta}(1+\tau)^{-\alpha\rho\eta}.$$

Since demands are independent of total public spending $S := X + Z$, they can alternatively be expressed as functions of (τ, T, Ω): using

$$X = \Omega S/(1+\Omega), \qquad \text{(A5a)}$$

$$Z = S/(1+\Omega) \qquad \text{(A5b)}$$

in (A4) yields the demand functions (22) in the main text.

Substituting (A4) back into (20) yields the indirect utility of composite consumption

$$\hat{Q}(\tau, T, X, Z) = (1-T)^\alpha \pi^{1/(\rho\eta)} \qquad \text{(A6)}$$

Again, using (A5) we can rewrite this as

$$\hat{Q} = \hat{Q}(\tau, T, \Omega, S) = (1-T)^\alpha \left[\frac{S}{1+\Omega}\right]^\delta \left[\frac{1-\beta}{1+\tau}\right]^{1/\rho} [\Delta + 1 + \tau]^{1/(\rho\eta)}. \qquad \text{(A7)}$$

A.3   Proofs of Propositions

Derivation of Equation (31) *in Proposition 3.* Since $\sigma > 1$ and $\alpha + \delta < 1/\rho$ by assumptions (20f) and (20g), we have $0 < \rho < 1$, $\eta > 1$ and thus $\delta\rho\eta < 1$. Hence, the RHS of (28) is convex and the LHS is linear.

Equation (28) has exactly one solution if its LHS describes a tangent to its RHS. Thus, denote by $\tau_{||}$ a tax rate for which the derivatives of the LHS and the RHS of (28) with respect to τ are equal, yielding

$$M - 1 = \Delta_\tau^*(\tau_{||}) = HR(1+\tau_{||})^{H-1}. \qquad \text{(A8)}$$

This can be solved for $\tau_{||}$, which equals the solution τ^* if the LHS and the RHS attain the same value: substituting $\tau_{||}$ back into (28) gives Condition (31) with equality.

Equation (28) has two solutions (no solution) if at the tax level $\tau_{||}$ for which the LHS and the RHS are parallel, the LHS is larger (smaller) than the RHS:

$$\tau_{||} > (<)\frac{1}{M}\left[\Delta^*(\tau_{||}) + 1 + \tau_{||}\right]. \tag{A9}$$

Using the solution of (A8) in this expression gives the upper (lower) case of Condition (31). □

Proof of Proposition 4. The derivation of Condition (33) is the same as for (28), see the proof of Proposition 3 above, so we abbreviate it here:

Again, the LHS of (32) is linear and the RHS is convex by (20f) and (20g). By comparing the first derivatives of the LHS and the RHS of (32),

$$1 \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \frac{n\phi}{m\alpha}\left[\Delta_\tau + 1\right], \tag{A10}$$

solving for τ and substituting the result back into (32), we obtain (33).

Furthermore, if there are two intersections of the LHS and the RHS of (32), the slope of the LHS must be larger than that of the RHS at the intersection with the lower τ value. Thus, (34) follows directly from the upper case of (A10).

Similarly, it follows from the 'middle case' of (A10) that (34) holds with equality if there is one solution. □

Proof of Proposition 5. We first show (35): Assume that there is a first-best τ^* satisfying (28). The RHS of (32) is smaller than the RHS of (28) at τ^*, since $\bar{\Omega} < \Omega^*$ implies $\Delta(\tau^*, \bar{\Omega}) < \Delta(\tau^*, \Omega^*)$.

Since the RHS of (32) by (20f) and (20g) monotonously increases in τ at a growing rate, and the LHS of (32) is the identity function as for (28), they must intersect at a smaller tax than for the first-best case: there must be a τ' that solves (32) with $\tau' < \tau^*$.

Since the RHS of (32) starts from $1/M[\Delta(\tau = 0, \bar{\Omega}) + 1] > 0$, and thus above the identity function, we also have $\tau' > 0$.

Relation (36) follows by the same logic, since a lower constraint on Ω implies a smaller Δ at the 'old' intersection. Formally, in the argument above, just replace τ' by $\tau'(\bar{\Omega}_2)$ and $\bar{\Omega}$ by $\bar{\Omega}_2$, as well as τ^* by $\tau'(\bar{\Omega}_1)$ and Ω^* by $\bar{\Omega}_1$. □

Proof of Proposition 6. We first show (38) and then that there are at most two non-zero solutions.

The proof of (38) has two parts: we show that the same relation holds for another specific $\Omega \neq \Omega'$, and then show that this implies the claim.

Define $\hat{\Omega}$ as the solution to

$$\Omega = \frac{\Delta(\bar{\tau}, \Omega)}{1 + \tau_p(\bar{\tau}, \Omega)}. \tag{A11}$$

In Fig. 2, this is the intersection of the linear LHS of (37) and the lowest curve described by the first fraction on the RHS of (37), the ratio of marginal total costs of clean and dirty consumption. For $\Omega \to 0$, this fraction goes to zero (because Δ goes to zero and τ_p towards a finite value), but its slope becomes infinitely large:

$$\lim_{\Omega \to 0} \frac{d}{d\Omega} \frac{\Delta}{1 + \tau_p} = \lim_{\Omega \to 0} \delta\rho\eta \frac{M + 1 + \bar{\tau}}{M(1 + \tau_p)^2} \frac{\Delta}{\Omega} = const. \cdot \lim_{\Omega \to 0} \Omega^{\delta\rho\eta - 1} = \infty,$$

since $\delta\rho\eta < 1$ by (20f) and (20g). Hence, it grows faster than (and is above) the LHS of (A11) near the origin. As Ω increases, it grows monotonously with a decreasing slope towards an upper bound:

$$\lim_{\Omega \to \infty} \frac{\Delta}{1 + \tau_p} \stackrel{\text{l'Hôpital}}{=} \lim_{\Omega \to \infty} \frac{\Delta_\Omega}{1/M\Delta_\Omega} = M. \tag{A12}$$

Thus,

$$\Omega < \frac{\Delta(\bar{\tau}, \Omega)}{1 + \tau_p(\bar{\tau}, \Omega)} \quad \Leftrightarrow \quad \Omega < \hat{\Omega}, \tag{A13}$$

and vice versa for $\Omega > \hat{\Omega}$, so (A11) has exactly one solution.

Now, to show that (38) holds for $\hat{\Omega}$, we need to show that the second term on the RHS of (37), in square brackets, is larger than one at $\Omega = \hat{\Omega}$, which is the case if

$$\tau_p(\bar{\tau}, \hat{\Omega}) > \bar{\tau}. \tag{A14}$$

By (25), such a relation holds for *any* Ω if $(M - 1)\bar{\tau} < 1$. Otherwise, we need another auxiliary construction:

Define $\tilde{\Omega}$ as the solution to

$$\tau_p(\bar{\tau}, \Omega) = \bar{\tau}, \tag{A15}$$

and note that since τ_p is an increasing function of Ω,

$$\tau_p(\bar{\tau}, \Omega) > \bar{\tau} \quad \Leftrightarrow \quad \Omega > \tilde{\Omega}. \tag{A16}$$

It follows that if $\tilde{\Omega} < \hat{\Omega}$, (A14) holds. We thus need to show that the left part of (A13) holds for $\tilde{\Omega}$.

To do so, note that (A15) by (25) implies $\Delta(\bar{\tau}, \tilde{\Omega}) = (M-1)\bar{\tau} - 1$. Hence, we can rewrite (A13) for $\Omega = \tilde{\Omega}$ as

$$\tilde{\Omega} < \frac{(M - 1)\bar{\tau} - 1}{1 + \bar{\tau}}. \tag{A17}$$

Substituting $\tilde{\Omega}$ with the help of (21) and reordering (using $(M-1)\bar{\tau} \geq 1$) yields

$$(M-1)\bar{\tau} - 1 < \left[\frac{\beta}{1-\beta}(1+\bar{\tau})^{1-\delta\rho} \right]^{\eta/(1-\delta\rho\eta)}. \tag{A18}$$

Using the definition of Δ^* from (28), we obtain

$$\bar{\tau} < \frac{1}{M}\left[\Delta^*(\bar{\tau}) + 1 + \bar{\tau} \right]. \tag{A19}$$

The RHS of (A19) is a convex function of $\bar{\tau}$ and always positive (see also Fig. 1 and the proof of Proposition 3). Thus, it is larger than the unity function on the LHS up to a potential intersection, which by (28) and Proposition 3 is the first-best τ^*: in this case our second-best assumption $\bar{\tau} < \tau^*$ implies that (A19) is true. If there is no such intersection, (A19) is trivially true (and there is no first-best solution).

Thus, we have shown that (38) holds for $\Omega = \hat{\Omega}$. It remains to be shown that this implies that it also holds for $\Omega = \Omega'$:

The second term on the RHS of (37) in the square brackets is larger than one by (A14) for $\Omega = \hat{\Omega}$, and it keeps growing monotonously with increasing Ω.[15] We saw above that the first term on the RHS of (37) also grows (towards an upper bound, see (A12)). Thus, if the RHS attains the same value as the LHS, this will be at a value $\Omega' > \hat{\Omega}$, and the term in the square brackets on the RHS will be larger than for $\hat{\Omega}$, so (38) from the main text follows.

Now, consider the number of solutions. Overall, the RHS of (37) grows faster than the LHS for $\Omega \to 0$, and also for $\Omega \to \infty$: Since the term in square brackets grows at a rate that converges to η/M, while the fraction in front of the bracket converges to a constant M, the slope of the entire RHS converges to η, which is larger than one (by $\sigma > 1$) and thus larger than the slope of the LHS. Nevertheless, the derivative of the RHS may fall below that of the LHS 'in between', for example if the growth of the first term on the RHS slows down fast (because this term then itself contributes less to overall growth of the RHS, and because it acts as a weight to the growth contribution of the second term in the square brackets, which will then be small). In this case, the RHS will have a 'curved' shape as in Figure 2: If growth of the RHS increases too fast after the slowdown, it does not have any point in common with the LHS, and there is no solution to (37). If the LHS is tangent to the RHS, there is only one solution. If the RHS has a first

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[15]More precisely, it grows to infinity at a monotonously decreasing slope that converges to η/M, since

$$\lim_{\Omega\to\infty} \frac{d}{d\Omega}\left[\frac{\tau_p - \bar{\tau}}{\tau_p}(1+\Omega) \right] = \lim_{\Omega\to\infty}\left[\underbrace{\frac{\tau_p - \bar{\tau}}{\tau_p}}_{\to 1} + \frac{\delta\rho\eta\bar{\tau}}{M}\frac{1+\Omega}{\Omega}\underbrace{\frac{\Delta}{\tau_p^2}}_{\to 0} \right] = 1.$$

intersection with the LHS at Ω'_L, the fact that the RHS becomes steeper afterwards until it reaches a slope larger than that of the LHS implies that there is a second solution at $\Omega'_H > \Omega'_L$ (see inset in Figure 2).

This concludes the proof. $\qquad\square$

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