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From agriculture to manufacture:
How does geography matter ?

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CORE

DISCUSSION PAPER

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**From agriculture to manufacture:
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Abstract

This paper shows that the development from an agricultural regime through industrialization to a manufacturing regime occurs simultaneously to the demographic transition and the change in labor structure towards an increasing fraction of skilled labor due to technological progress. The manufacturing sector is economically viable when the technological level is sufficiently high. During the industrialization, the technological progress makes technology become more complementary to skilled labor than to unskilled labor, so that individuals tend to decrease the number of unskilled offspring in order to increase the number of skilled ones. This paper also shows that a geographical advantage for agriculture helps an economy to be more prosperous in the agricultural regime, but delays the timing of industrialization and the timing of demographic transition. Hence, an economy with more geographical advantage for agriculture may be overtaken in the development process by another with less geographical advantage for agriculture when the level of technology is high enough.

Keywords: agricultural sector, manufacturing sector, technological level, technological progress, geographical advantage for agriculture.

JEL classification: J11, J13, O11, O41.

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1 Introduction

The development process from agriculture to manufacture along with the demographic transition and the different timings of industrialization, which has led to the divergence across countries, are interesting topics in development economics that have been subject of intensive research in recent years. This paper aims at contributing a mechanism helping to explain the development from the agricultural regime to the manufacturing regime occurring along with the demographic transition and the change in labor structure towards an increasing fraction of skilled labor. Moreover, we highlight the role of geographical advantages for agricultural production in shaping the patterns of economic development, contributing to the divergence and reversals in economic performance across societies. We show that when the technological level is low enough, only the agricultural production is operative. Over time, when the technological level is sufficiently high, the manufacturing production becomes economically viable, and the fraction of skilled labor increases along with the accumulation of technology. This paper sheds light on the fact that a geographical advantage for agricultural production helps an economy to be more prosperous in the agricultural regime but delays its industrialization and demographic transition. Moreover, the economic performances may be reversed in the development process between countries with more geographical advantage for agriculture and those with less geographical advantage.

The closest literature to this paper may be Ashraf and Galor (2012), “*Cultural Diversity, Geographical Isolation, and the Origin of the Wealth of Nations*”, and Galor and Mountford (2006), “*Trade and the Great Divergence: The Family Connection*”. This paper, however, differs significantly from Ashraf and Galor (2012) at least in two fundamental aspects: (i) Ashraf and Galor (2012) considers the exogenous effects of geographical isolation on cultural diversity affecting the creation of knowledge to explain the asymmetric evolution across societies, while this paper explains the asymmetric evolution across societies by considering the entire geographical environment and resources which are advantageous for agricultural production; (ii) Ashraf and Galor (2012) does not take into account the demographic transition in explaining the development process across societies, while this paper does. This paper also differs significantly from Galor and Mountford (2006) at least in two aspects: (i) By not taking into account the role of geography, Galor and Mountford (2006) shows international trade to be a prime cause of the Great Divergence in per capita income across countries in the last two centuries, while here we argue that geography is a deeper fundamental cause; (ii) While this paper explains the switching from agricultural production to manufacturing production, along with demographic transition, due to technological progress during the development process, Galor and Mountford (2006) does not.

The two last centuries are characterized by the significant technological progress associated with the industrial revolution, the demographic transition, and the generalization of basic education. As a consequence, most societies got out of Malthusian stagnation and experienced a considerable increase in the income per capita and human capital, as well as a decline in the growth rate of population as depicted in the Figure 1 for the case of US. The different timings of the transition from agricultural production to manufacturing production across societies have shaped considerably the contemporary world economy (as depicted in the Figure 2) and need to be explained.

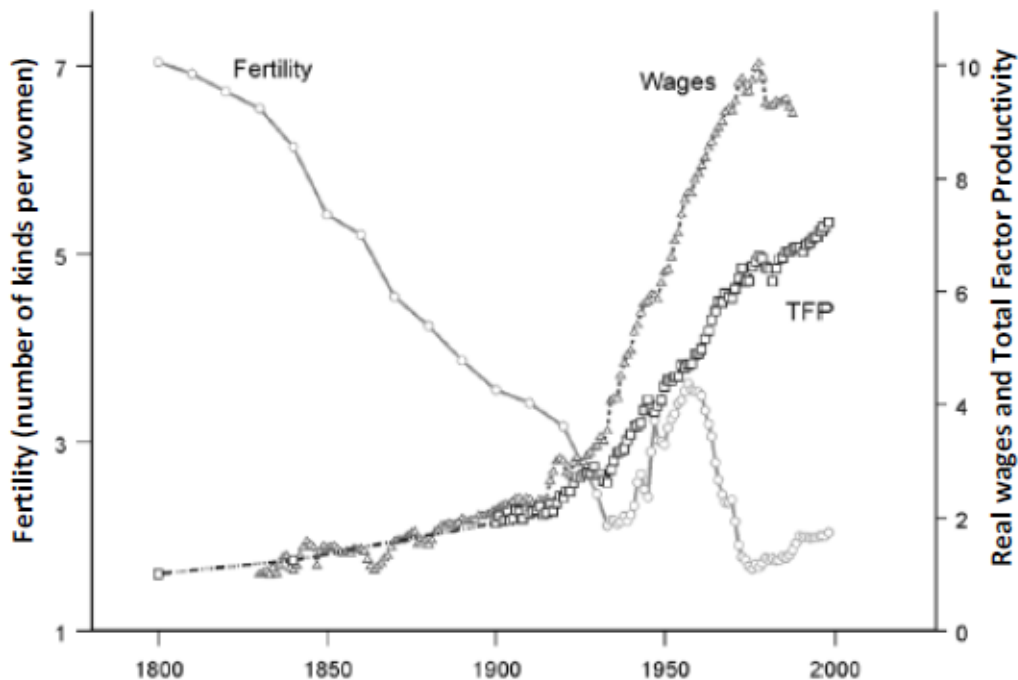


Figure 1. Technological progress, fertility, and return to labor; US 1800 - 2000. Quoted in Greenwood and Seshadri (2005)

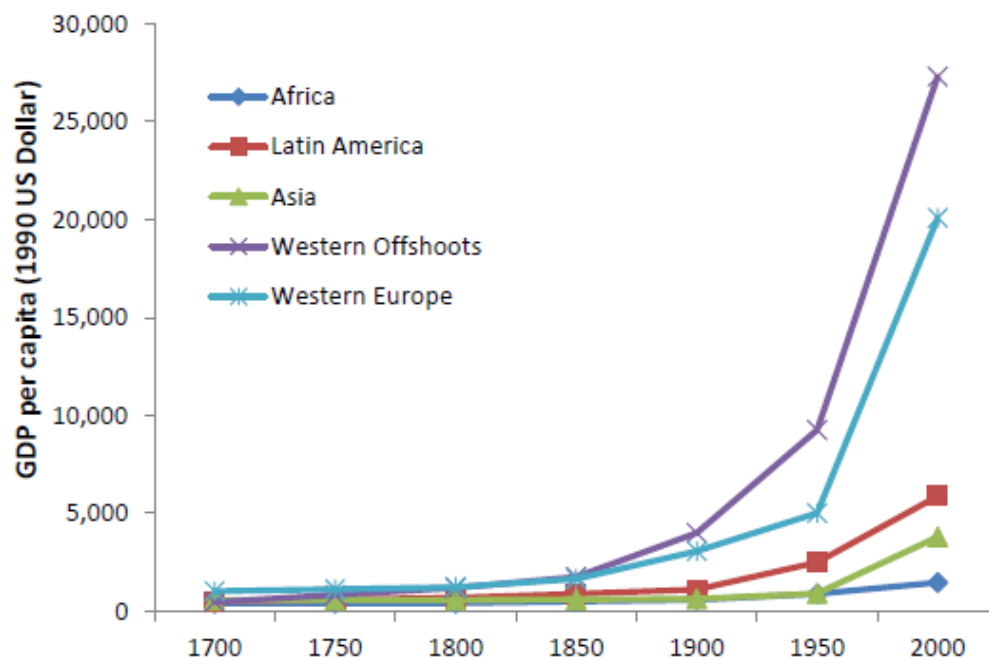


Figure 2. The differential timings of the take-off across regions. Source: Maddison (2003)

The divergence across societies in the two last centuries is also marked by the divergence in the per capita level of industrialization (measuring per capita volume of industrial production) which can also be explained in this paper.

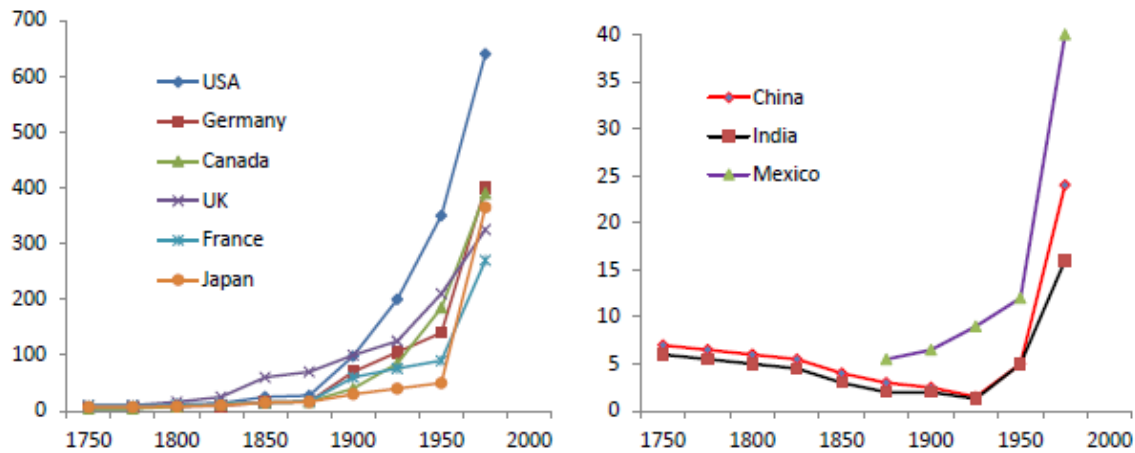


Figure 3. Per capita levels of industrialization (UK in 1900 = 100). Source: Bairoch (1982)

The industrial revolution in Western Europe is reflected by the sharp acceleration in process of urbanization, as depicted in the Figure 4, which proxies the change in the structure of labor towards an increasing proportion of manufacturing workers.

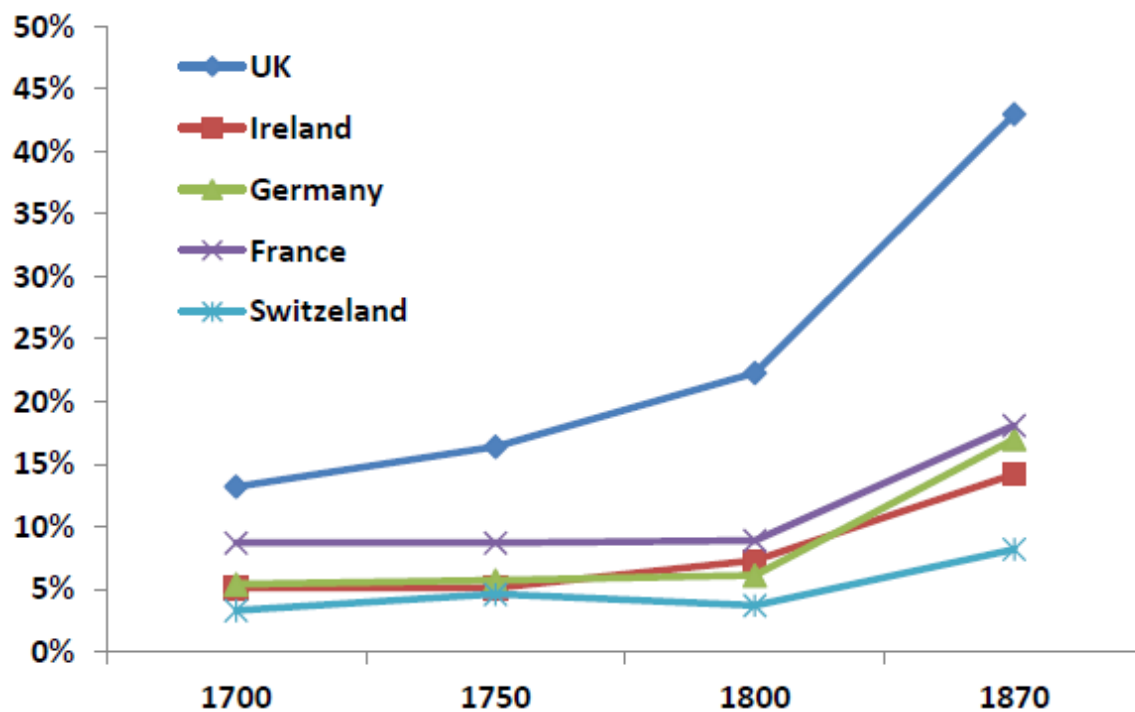


Figure 4. Urbanization rate (percentage of city with population larger than 10,000) in Western Europe: 1700–1870. Sources: Bairoch (1988) and De Vries (1984).

Historical evidence shows that the wealth of Asia, particularly China and India, was ahead of Europe until 11th century. Nonetheless, Asia had been already overtaken by the time of the Industrial Revolution in the 18th century and beyond. This story is interesting and can be explained in this paper.

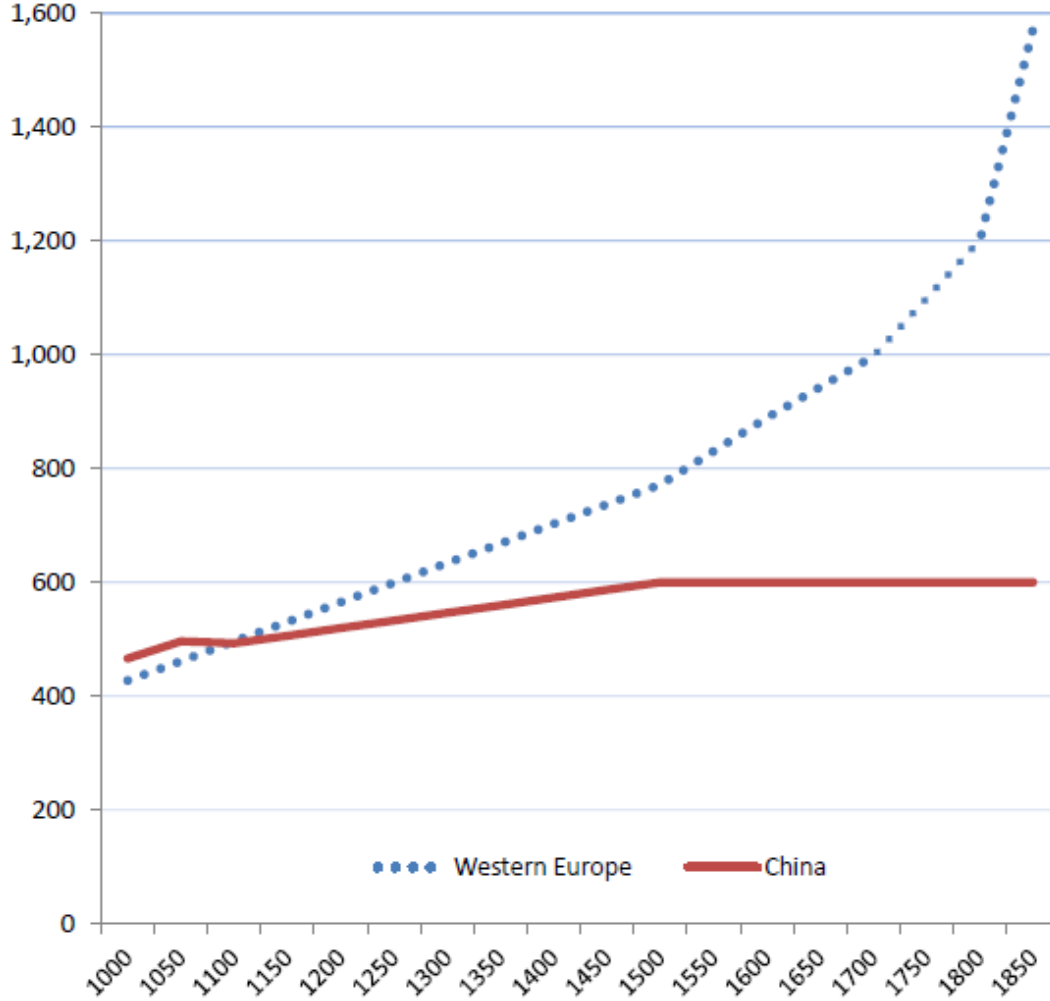


Figure 5. Comparative levels of GDP per capita (1990 US Dollar): China and Western Europe, 1000 - 1850 A.D. Source: Maddison (2003)

This paper extends Galor and Mountford (2006) by introducing a geographical advantage for agricultural production to explain the historical facts depicted in Figures 1 to 5 above. The rest of the paper is organized as follows. Section 2 reviews the literature. Section 3 introduces the benchmark model. Section 4 analyzes the equilibrium in the labor market. The effects of technology on fertility and labor structure are analyzed in section 5. The competitive equilibrium and dynamical system are presented in section 6. Section 7 discusses the role of geography and a tale of two countries. Section 8 concludes the paper.

2 Literature Review

There is a huge literature in the research fields of this paper. In this section, we try to review two strands of economic growth and development literature which relate to this paper: (i) the emergence of human capital and the demographic transition in unified growth models, and (ii) the causes of economic growth and the great divergence.

2.1 Human capital and demographic transition in unified growth models

The role of human capital formation on the demographic transition leading to the great divergence across societies has been emphasized in the unified growth theories advanced by Galor and Weil (2000) and Galor and Moav (2002), and confirmed empirically by Glaeser et al. (2004), and recently by Becker and Woessman (2009) and Becker et al. (2010, 2011). More recently, Cervellati and Sunde (2013) also develop a unified growth model to study the mechanism generating economic growth and the demographic transition as well as development. These papers, in general, show theoretically and quantitatively that the increase in technologically-driven demand for human capital and its effect on educational investment play a central role in the transition from Malthusian stagnation, through the demographic transition, to modern sustained economic growth. Doepke (2004) advances a unified growth model to examine whether government policies on education and child labor can account for cross-country variations in fertility decline. He shows that education subsidies have only minor effects, while accounting for child labor regulation is important. Dao (2013) also proposes a mechanism linking technology, gender inequality, and fertility in a unified growth model to explain the demographic transition accompanied with accelerated growth. Dao (2013) highlights the role of technological progress not only in freeing women from housework, but also in improving the equality in human capital between males and females, hence contributing significantly to the demographic transition during development process.

Hansen and Prescott (2002) advance a two-sector and one-good overlapping generations model, close to our model, to explain the transition from Malthusian technology to Solow technology along with the change in labor structure (the fraction of labor in the Solow sector) and the demographic transition. The model in this paper, however, differs from the one in Hansen and Prescott (2002) in some fundamental aspects: (i) our model considers a fertility choice in the utility maximization of the individual, while in Hansen and Prescott (2002), fertility is a consequence of consumption by young households; (ii) Hansen and Prescott (2002) does not take into account the heterogeneity of children, while our model does; and (iii) in Hansen and Prescott (2002), technological progress in both sectors is exogenous, while our model endogenizes technological progress.

2.2 The causes of economic growth and the great divergence

What triggered economic growth, and why some countries are so rich and some others so poor are controversial topics in the literature of economic growth. According to Acemoglu (2009, p.109), any explanation for the differences in income per capita based on the differences across countries in technology, physical capital, as well as human capital is incomplete. He argues that if technology, physical capital, and human capital are so important in understanding differences in the wealth of nations, then *why do not some countries improve their technology, accumulate physical capital, and invest in human capital as much as others do?* So there must be other and deeper reasons that are the fundamental causes of economic growth.

So what could these fundamental causes be? Innumerable causes for economic growth have been proposed in the literature by economists, historians, and social scientists. Acemoglu (2009) classifies the main hypotheses into four categories: (i) the luck hypothesis, (ii) the culture hypothesis, (iii) the institutions hypothesis, and (iv) the geography hypothesis. He refers by the “luck” hypothesis to the set of causes leading to divergence in economic performances between countries that are otherwise identical. They can be just different selections among multiple equilibria. He mentions that “luck” explanations are often theoretically

grounded. The empirical plausibility of this hypothesis is another matter. The influences of geographical, cultural, and institutional factors have been at the center of the debate on the explanation of the remarkable transformation of the world income distribution over last two centuries.

The significant influence of cultural factors on economic development of Europe and Asia is originally pioneered by Weber (1905, 1922), and promoted later by Hall (1986), Lal (1998), and Landes (1998, 2006), among others.² According to Weber, the origins of the industrialization in Western Europe can be traced to the Protestant reformation and particularly the rise of Calvinism. Ashraf and Galor (2012) state that: “the Weberian viewpoint places the proclivity of European culture towards rationalism and the objective “*disenchantment of the world*” at the forefront in explaining the rise of industry in the Western world”. Indeed, his ideas have been proposed as explanations for why Latin American countries are relatively poor (because of their Iberian culture), while North American countries are more prosperous (because of their Anglo-Saxon culture), (see Acemoglu 2009, p.122). However, according to Acemoglu, there are two challenges to the theories of economic growth based on culture: (i) the difficulty of measuring culture; and (ii) the accounting for miracles, such as those of Asian Tigers³. If Asian cultural values are crucial for the miracles of these Asian countries, then why these values did not lead to growth before? If these cultural values are crucial for the miracle of South Korea then why they do not spur growth in North Korea?, etc. Acemoglu (2009) argues that these challenges are solvable if we take into account the role of institutions. In this light, hence, culture may be viewed as a complement to institutional factors.

The influence of institutions was first emphasized by North and Thomas (1973), and then advanced by North (1981), Landes (1998), and others, arguing that institutional factors, which facilitate the property rights and enhance technological progress as well as the diffusion of knowledge, were fundamental factors in the early European transition to sustained economic growth and the great divergence across the globe. Recently, Acemoglu et al. (2005) show that economic outcomes are shaped by economic institutions through the incentives and constraints they impose on economic agents. There is generally a conflict over social choices because different groups and individuals typically benefit differently from different economic institutions. Such a conflict is ultimately solved in favor of groups with greater political power. The political power of each group is determined by political institutions and the distribution of resources. They argue that: “Economic institutions encouraging economic growth emerge when political institutions allocate power to groups with interests in broad-based property rights enforcement, when they create effective constraints on power-holders, and when there are relatively few rents to be captured by power-holders”. So, differences in political institutions lead to differences in economic performance, and hence lead to divergence across societies.

The effects of geography on economic growth and divergence between societies have been emphasized recently by Jones (1981), Diamond (1997), Gallup, Sachs and Mellinger (1998), and Ashraf and Galor (2012), and others. The geography hypothesis, first and foremost, is the fact that not all regions of the world are equally apt for living and production. Nature, that is, the ecological and geographical environment of nations may play a major role in their

²As mentioned in Ashraf and Galor (2011), the term “culture” refers to the set of society’s norms, beliefs, customs, traditions, taboos, codes of conduct, etc., and it is therefore distinct from the notion of “institutions”, which has been regarded in the literature as embodying the sociopolitical environment determined by constitutions, laws, and property rights.

³Asian Tigers refer to South Korea, Singapore, Taiwan, and Hong Kong

economic experiences. There are at least three main branches of geography hypothesis, each emphasizing different mechanism for how geography affects prosperity. The first one, and also the earliest one, is proposed by Montesquieu in 1748. He believed that climate, in particular heat, shaped human attitudes and effort, and through this channel, it affects both economic and social outcomes. The second one, which is developed by Gunnar Myrdal, emphasizes the impact of geography on the technologies available to a society, especially in agriculture. Myrdal (1968, p.2121) wrote: “*Serious study of the problems of underdevelopment should take into account the climate and its impacts on soil, vegetation, animals, humans and physical assets - in short, on living conditions in economic development*”. The third variant of the geography hypothesis, which is proposed by Jefferey Sachs, links poverty in many areas of the world to their disease burden, emphasizing that “the burden of infectious disease is higher in the tropics than in the temperate zones” (Sachs, 2001). In this paper, the geographical factor is likely closer to the second version of geography hypothesis.

In the widely popular book “*Guns, Germs, and Steel: The Fates of Human Societies*”, Jared Diamond (1997) provides a historical account along with research results from other sciences such as biology, geography, archeology, epidemiology, etc., to explain why the world becomes so unequal across communities. And why some regions, peoples, and cultures developed more quickly than others. Diamond pushes the series of causes and consequences back to 13,000 years ago to reach the conclusion that the origin of the great divergence is due to initial differences in geographical and biological conditions. Like espousing the second view of geography hypothesis, he argues that geographical differences between Americas and Eurasia determined the timing and nature of settled agriculture and, by means of this channel, shaped whether societies were able to develop complex organizations and advanced civilian and military technologies. Although whether the study of human societies can be pursued as scientifically is still a controversy, the work of Diamond depicts the most general picture of human history during the last 13,000 years and shows a way for other sciences to develop theories of development.

Ashraf and Galor (2012) provide a mechanism linking geographical isolation to cultural diversity, affecting the creation and accumulation of knowledge, to explain the asymmetric evolutions across societies. They show that societies characterized by less geographical vulnerability to cultural diffusion benefited from enhanced assimilation, lower cultural diversity, and more intense accumulation of society-specific human capital. So, these societies were more efficient with respect to their production-possibility frontier and flourished in the agricultural stage of development. However, the lack of cultural diffusion diminished the ability of these societies to adapt to new technological paradigms, which delayed their industrialization and, hence, their take-off to a sustained growth regime. Their empirical analysis shows that (i) geographical isolation prevalent in pre-industrial period has had a persistent negative impact on the extent of contemporary cultural diversity; (ii) pre-industrial geographical isolation had a positive impact on economic development in the agricultural stage but has had a negative impact on income per capita in the course of industrialization; and (iii) cultural diversity has had a positive impact on economic development in the industrialization process.

Other approaches to explain the asymmetric evolutions across the globe are based on the roles of ethnic, linguistic, religious fractionalization, and human genetic diversity. The effects of ethnolinguistic fractionalization on comparative development across societies are examined empirically by Easterly and Levine (1997) and Alesina et al. (2003). These papers have demonstrated that geopolitical factors, which brought a high degree of ethnic fractionalization

in some regions of the world, led to the implementation of poor institutions and, hence, to a divergence in the development process across societies. Ashraf and Galor (2013) advance and empirically establish a hypothesis that, in the prehistoric course of *Homo-sapiens* out of Africa, the variation in migratory distance to various destinations across the globe has affected the genetic diversity and has had a persistent hump-shaped effect on comparative economic development. Ashraf and Galor (2013) show that the low diversity of Native American and the high diversity of African have been detrimental for the development process of these regions, while the intermediate levels of diversity of European and Asian have been conducive for their development process.

3 The benchmark model

In this paper, we focus on the geography hypothesis proposed by Myrdal (1968) as mentioned in the literature review section, i.e. we consider the impact of geography on the technology available to a society, especially in agriculture. Of course, geography is not everything in explaining the asymmetric development across countries, but from previous research in the literature, geography seems to play a crucial role, at least, in the early stages of development.

We consider competitive overlapping generations economies in the process of development with two sectors of production, agriculture and manufacture, and one final output. We assume that the factors of production in the agricultural sector are land and unskilled labor, and the only factor of production in the manufacturing sector is skilled labor. In any period, each individual in the economy chooses a number of skilled children and unskilled children as well as consumption, under the budget constraint, so as to maximize his/her utility.

3.1 Environmental resources for agriculture

We refer by “land”, which is absent in Galor and Mountford (2006), to the entire geographical environment and natural resources of the economy supporting agricultural production. Due to technological constraint, an economy may not make the most of the available “land” (i.e. entire natural resources and geographical environment) for agricultural production, e.g. people may just occupy the part of their territory and resources that are the most suitable for agriculture. This “part of land” is called “productive land” whose size in period t , X_t , is determined by

$$X_t = \chi(A_t)X \quad (1)$$

where $\chi(A_t) \in (0, 1)$ is the level of accessibility to the “land” depending on the level of technology in period t , A_t ; $\chi'(A_t) > 0$, $\chi''(A_t) < 0$; X is the entire natural resources and geographical environment for agricultural production. Without loss of generality, we normalize $X = 1$ for simplicity. Hereafter, therefore, the natural resources and geographical environment available for agricultural production in period t is $X_t = \chi(A_t)$.

3.2 Production

In every period t , output can be produced in the agricultural sector and/or in the manufacturing sector. The agricultural sector uses unskilled labor and “land” as factors of production to produce output according to Cobb-Douglas production technology

$$Y_t^a = \chi(A_t)^\alpha (L_t^u)^{1-\alpha}; \quad \alpha \in (0, 1) \quad (2)$$

where L_t^u is the aggregate amount of unskilled labor of the economy in period t .

Indeed, the way the use of “land” for agricultural production is modeled here differs from that in Ashraf and Galor (2012) by making it depend on technology. In Ashraf and Galor (2012), land for agricultural production is always fixed at $X = 1$ regardless of the level of technology. That paper, however, introduces a level of technology specific for agriculture whose dynamics can be traced to depend on the geographical isolation of the economy. That is to say Ashraf and Galor (2012) consider the effect of a geographical factor differing from that in this paper for agricultural production.

We assume that, in the context of an early stage of development, there is no property rights to “land” (i.e. geographical environment and resources), so that the return to land is zero⁴ and the return to labor, or *the inverse demand for labor*, in the agricultural sector in period t is

$$w_t^u = \left(\frac{\chi(A_t)}{L_t^u} \right)^\alpha \quad (3)$$

Similar to Ashraf and Galor (2012), the output of the manufacturing sector in period t is determined by a linear, constant returns to scale production technology such that

$$Y_t^m = A_t L_t^s \quad (4)$$

where L_t^s is the aggregate amount of skilled labor of the economy in period t . The return to labor, or *the inverse demand for labor*, in the manufacturing sector in period t is

$$w_t^s = A_t \quad (5)$$

The total labor force of the economy in period t is

$$L_t = L_t^u + L_t^s \quad (6)$$

3.3 Technology

The dynamics of technological level is defined by

$$A_{t+1} = (1 + g_t)A_t \quad (7)$$

where g_t is the rate of technological progress between periods t and $t + 1$. We assume that g_t depends on the average skill of labor in the economy, i.e. it is determined by the fraction of skilled labor in the economy. In particular,

$$g_t = g(h_t)$$

where $h_t = L_t^s/L_t$ is the fraction of skilled labor over the total labor force of the economy in period t ; $g(h) > 0$, $g'(h) > 0$, $\forall h \geq 0$.

3.4 Individuals

Basically, the individual’s problem in this paper follows from Galor and Mountford (2006). We use superscripts “ u ” and “ s ” to denote for unskilled and skilled labor respectively. In

⁴We can justify the assumption of “no property rights to land” as: (i) “land” in our model stands for the entire geographical environment and resources then basically “land” is collectively owned with the proceeds distributed as a lump sum to the population; (ii) we consider a completely aggregate economy, so the model disregards the heterogeneity across households, say labourers and “landlords”; in addition, (iii) although there is no property right to these resources, the economy cannot engage in the over-exploit action of common resources known as the “tragedy of commons” because the accessibility to these resources is constrained by the technological level through the function $\chi(A)$.

every period $t \in \mathbb{N}$, a generation consists of L_t^u identical unskilled individuals working in the agricultural sector and L_t^s identical skilled individuals working in the manufacturing sector. Individuals live for two periods. In the first period, say childhood, they consume a fraction of their parent's time. In the second period, say adulthood, they allocate their time endowment between child-rearing and labor force participation.

As in Galor and Mountford (2006), the utility of an adult $i \in \{s, u\}$ in period t comes from consumption and the total potential income of his/her children. In particular,

$$u_t^i = \gamma \ln(w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i}) + (1 - \gamma) \ln c_t^i \quad (8)$$

where $n_t^{s,i}$ and $n_t^{u,i}$ are the number of children to be skilled and unskilled workers respectively; w_{t+1}^s and w_{t+1}^u are their wages in period $t + 1$ respectively; c_t^i is total amount of consuming agricultural goods and manufacturing goods by individual i . We assume here these two kinds of good are perfectly substitutable.

The budged constraint of an adult i in period t is

$$c_t^i + w_t^i(\phi^s n_t^{s,i} + \phi^u n_t^{u,i}) \leq w_t^i \quad (9)$$

where ϕ^s and ϕ^u are costs in time to raise one offspring to be a skilled worker and a unskilled worker, respectively. We assume that $\phi^s > \phi^u$, implying raising a skilled child is more costly than raising an unskilled one.

An adult individual $i \in \{s, u\}$ in period t chooses the number of skilled and unskilled children and consumption under the budget constraint (9) so as to maximize his/her utility (8), i.e.

$$\max_{c_t^i > 0; n_t^{s,i}, n_t^{u,i} \geq 0} \gamma \ln(w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i}) + (1 - \gamma) \ln c_t^i$$

subject to

$$c_t^i + w_t^i(n_t^{s,i} \phi^s + n_t^{u,i} \phi^u) \leq w_t^i$$

Solving the individual's problem above (see Appendix A1), we have

$$c_t^i = (1 - \gamma) w_t^i \quad (10)$$

$$n_t^{s,i} \phi^s + n_t^{u,i} \phi^u = \gamma \quad (11)$$

where

$$\begin{aligned} n_t^{s,i} = 0, n_t^{u,i} = \frac{\gamma}{\phi^u} & \quad \text{if} \quad \frac{w_{t+1}^s}{\phi^s} < \frac{w_{t+1}^u}{\phi^u} \\ n_t^{s,i} = \frac{\gamma}{\phi^s}, n_t^{u,i} = 0 & \quad \text{if} \quad \frac{w_{t+1}^s}{\phi^s} > \frac{w_{t+1}^u}{\phi^u} \\ n_t^{s,i} > 0 \quad \text{and} \quad n_t^{u,i} > 0 & \quad \text{only if} \quad \frac{w_{t+1}^s}{\phi^s} = \frac{w_{t+1}^u}{\phi^u}. \end{aligned}$$

It is obvious from the homotheticity of preferences that the fertility and the composition between skilled and unskilled children do not depend on the income of their parent. Hence, hereafter in every period t , we remove the superscript i from variables $n_t^{s,i}$ and $n_t^{u,i}$ to write as n_t^s and n_t^u respectively.

To guarantee the population never collapses, we assume that

$$\frac{\gamma}{\phi^s} \geq 1 \quad (A1).$$

The assumption (A1) guarantees the growth rate of population is always non-negative. This assumption is rather consistent with the reality which is observed widely that population grows in most countries.

We know from (3) that the return to unskilled labor in period $t + 1$ depends on the size of unskilled population, L_{t+1}^u . And it will be apparent in section 4 (in Corollary 1) that the viability of the manufacturing sector in period $t + 1$ depends on the size of population, L_{t+1} . When the manufacturing sector is viable in period $t + 1$, the individuals will be indifferent between raising skilled and unskilled children when it holds $\frac{w_{t+1}^s}{\phi^s} = \frac{w_{t+1}^u}{\phi^u}$. That is to say the choices of numbers of skilled children and unskilled children of individuals in period t depend on the expected structure of labor in period $t + 1$. We assume that individuals in period t has a perfect foresight on the structure of labor in period $t + 1$, h_{t+1}^e . Hence, in the perspective of individuals, the choices of skilled and unskilled children are determined by

$$n_t^s \phi^s + n_t^u \phi^u = \gamma$$

and

$$\frac{n_t^s}{n_t^s + n_t^u} = h_{t+1}^e$$

i.e.

$$\begin{aligned} n_t^s &= \frac{\gamma h_{t+1}^e}{(\phi^s - \phi^u) h_{t+1}^e + \phi^u} \equiv n^s(h_{t+1}^e) \\ n_t^u &= \frac{\gamma(1 - h_{t+1}^e)}{(\phi^s - \phi^u) h_{t+1}^e + \phi^u} \equiv n^u(h_{t+1}^e) \end{aligned}$$

4 Equilibrium in the labor market

It is a fact that, as it will be apparent later, in the early stages of development, when the technological level is sufficiently low so that the labor productivity of the manufacturing sector is low relative to that of the agricultural sector, then output is produced only by the agricultural sector. However, in later stages of development, when the technological level is high enough, increasing the labor productivity of the manufacturing sector, then the manufacturing sector is economically viable.

The inverse demand for labor in the agricultural sector, as in (3), increases without bound when employment in that sector decreases. This implies that in a closed economy the agricultural sector is operative in every period. In contrast to the agricultural sector, the manufacturing sector is operative if, and only if the productivity in this sector is high enough. Proposition 1 and Corollary 1 below state the condition under which the manufacturing sector is economically viable.

Proposition 1: *In any period t , there exists a unique threshold of technological level, $\hat{A}_t = \hat{A}(L_t)$, such that the manufacturing sector is operative if, and only if:*

$$A_t \geq \hat{A}(L_t) \quad \text{where} \quad \hat{A}'(L_t) < 0$$

Proof: It follows from (3), (5), and the individual's optimization problem in period $t - 1$ that:

- (i) If $\frac{A_t}{\phi^s} > \left(\frac{\chi(A_t)}{L_t^u}\right)^\alpha / \phi^u$ then $n_{t-1}^s > 0$, and
- (ii) $n_{t-1}^s > 0$ only if $\frac{A_t}{\phi^s} \geq \left(\frac{\chi(A_t)}{L_t^u}\right)^\alpha / \phi^u$.

Now we prove that

$$n_{t-1}^s \begin{cases} > 0 & \text{if } \frac{A_t}{\phi^s} > \left(\frac{\chi(A_t)}{L_t}\right)^\alpha / \phi^u \\ = 0 & \text{if } \frac{A_t}{\phi^s} \leq \left(\frac{\chi(A_t)}{L_t}\right)^\alpha / \phi^u \end{cases} \quad (1)$$

(Note that in the denominators there is L_t instead of L_t^u).

The statement (1) can be rewritten as

- (1) If $\frac{A_t}{\phi^s} > \left(\frac{\chi(A_t)}{L_t}\right)^\alpha / \phi^u$ then $L_t^u < L_t$ (i.e. $L_t^s > 0$ or $n_{t-1}^s > 0$).

We prove (1) by a contradiction. In effect, if $L_t^u = L_t$ (i.e. $L_t^s = 0$ or $n_{t-1}^s = 0$) then

$$\frac{A_t}{\phi^s} > \left(\frac{\chi(A_t)}{L_t}\right)^\alpha / \phi^u$$

is equivalent to

$$\frac{A_t}{\phi^s} > \left(\frac{\chi(A_t)}{L_t^u}\right)^\alpha / \phi^u$$

which implies (from (i) above) that $n_{t-1}^s > 0$, leading to a contradiction with $n_{t-1}^s = 0$.

The statement (2) can be rewritten as

- (2) If $\frac{A_t}{\phi^s} \leq \left(\frac{\chi(A_t)}{L_t}\right)^\alpha / \phi^u$ then $L_t^u = L_t$ (i.e. $L_t^s = 0$ or $n_{t-1}^s = 0$).

We prove (2) by establishing its negation. In effect, if $L_t^u < L_t$ (i.e. $L_t^s > 0$ or $n_{t-1}^s > 0$) then, from (ii), it holds

$$\frac{A_t}{\phi^s} \geq \left(\frac{\chi(A_t)}{L_t^u}\right)^\alpha / \phi^u.$$

Moreover, since $L_t^u < L_t$ then $\left(\frac{\chi(A_t)}{L_t^u}\right)^\alpha / \phi^u > \left(\frac{\chi(A_t)}{L_t}\right)^\alpha / \phi^u$, from which we have

$$\frac{A_t}{\phi^s} > \left(\frac{\chi(A_t)}{L_t}\right)^\alpha / \phi^u$$

which is the negation of the condition $\frac{A_t}{\phi^s} \leq \left(\frac{\chi(A_t)}{L_t}\right)^\alpha / \phi^u$.

Hence, the manufacturing sector is economically viable in period t (i.e. $n_{t-1}^s > 0$) if, and only if, the marginal return to labor in that sector per unit of time raising skilled labor, A_t/ϕ^s , is higher than in the agricultural sector per unit of time raising unskilled labor, $\left(\frac{\chi(A_t)}{L_t}\right)^\alpha/\phi^u$, when the entire labor force is employed in the agricultural sector, i.e.

$$\frac{A_t}{\phi^s} > \frac{\left(\frac{\chi(A_t)}{L_t}\right)^\alpha}{\phi^u} \quad \Leftrightarrow \quad \frac{A_t}{\chi(A_t)^\alpha} > \frac{\phi^s}{\phi^u} L_t^{-\alpha}$$

Let $\psi(A_t) = \frac{A_t}{\chi(A_t)^\alpha}$, which is monotonically increasing in A_t . In effect,

$$\psi'(A_t) = \frac{\chi(A_t) - \alpha A_t \chi'(A_t)}{\chi(A_t)^{\alpha+1}} > 0$$

since $\chi(A_t) \in (0, 1)$ is strictly increasing concave and $\alpha \in (0, 1)$, it holds $\chi(A_t) - \alpha A_t \chi'(A_t) > 0$. Moreover,

$$\psi(0) = 0 \quad \text{and} \quad \lim_{A_t \rightarrow +\infty} \psi(A_t) = +\infty$$

Therefore, given L_t , there exists a unique \hat{A}_t such that

$$\frac{\hat{A}_t}{\chi(\hat{A}_t)^\alpha} = \frac{\phi^s}{\phi^u} L_t^{-\alpha} \tag{12}$$

By applying the implicit function theorem, we have

$$\hat{A}_t = \hat{A}(L_t) \quad \text{where} \quad \hat{A}'(L_t) = \frac{-\alpha \phi^s}{L_t^{1+\alpha} \phi^u} \frac{\chi(\hat{A}_t)^{1+\alpha}}{\chi(\hat{A}_t) - \alpha \hat{A}_t \chi'(\hat{A}_t)} < 0$$

Moreover, it is straightforward that

$$\lim_{L_t \rightarrow 0^+} \hat{A}(L_t) = +\infty \quad \text{and} \quad \lim_{L_t \rightarrow +\infty} \hat{A}(L_t) = 0$$

Q.E.D.

Corollary 1: *Given $A_t > 0$, there always exists a unique threshold size of population, \hat{L}_t , such that the manufacturing sector is operative in period t if, and only if*

$$L_t \geq \hat{L}_t = \left(\frac{\phi^s}{\phi^u}\right)^{1/\alpha} \frac{\chi(A_t)}{A_t^{1/\alpha}}.$$

Proof: It is straightforward from the necessary and sufficient conditions for the manufacturing sector to be operative, i.e., $A_t/\phi^s \geq \left(\frac{\chi(A_t)}{L_t}\right)^\alpha/\phi^u$. Q.E.D.

The Figure 6 below presents the equilibrium in the labor market and shows intuitively when the manufacturing sector is viable.

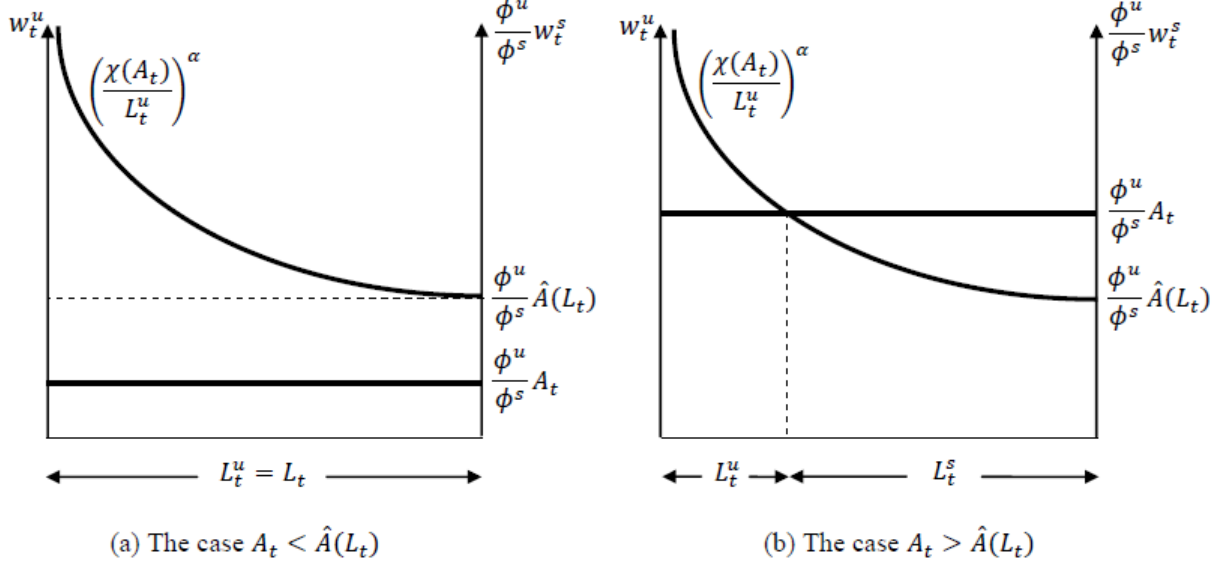


Figure 6. The equilibrium in the labor market

5 Impact of technology on fertility and the composition of labor

Before examining the dynamical system of the economy in the next section, it is interesting to analyze in this section the impact of technological progress on fertility and the structure of labor. These analyses will help us to understand better the simultaneous evolution of fertility and the structure of labor along with technology.

When the manufacturing is viable, then at the equilibrium it holds

$$\frac{w_t^s}{\phi^s} = \frac{w_t^u}{\phi^u}$$

i.e.
$$\frac{A_t}{\phi^s} = \frac{\left(\frac{\chi(A_t)}{L_t^u}\right)^\alpha}{\phi^u} \Leftrightarrow L_t^u = \left(\frac{\phi^s}{\phi^u}\right)^{1/\alpha} \frac{\chi(A_t)}{A_t^{1/\alpha}} \quad (13)$$

So the fraction of skilled labor is determined by

$$h_t = 1 - \frac{L_t^u}{L_t} = \begin{cases} 0 & \text{if } A_t \leq \hat{A}(L_t) \\ 1 - \left(\frac{\phi^s}{\phi^u}\right)^{1/\alpha} \frac{\chi(A_t)}{A_t^{1/\alpha} L_t} & \text{if } A_t \geq \hat{A}(L_t) \end{cases} \equiv h(A_t, L_t) \quad (14)$$

We have

$$\frac{\partial h_t}{\partial A_t} = \begin{cases} 0 & \text{if } A_t < \hat{A}(L_t) \\ \left(\frac{\phi^s}{\phi^u}\right)^{1/\alpha} \frac{\frac{1}{\alpha} \chi(A_t) - \chi'(A_t) A_t}{A_t^{\frac{1+\alpha}{\alpha}} L_t} > 0 & \text{if } A_t > \hat{A}(L_t) \end{cases}$$

and

$$\lim_{A_t \rightarrow +\infty} h_t = \lim_{A_t \rightarrow +\infty} \left[1 - \left(\frac{\phi^s}{\phi^u} \right)^{1/\alpha} \frac{\chi(A_t)}{A_t^{1/\alpha} L_t} \right] = 1$$

We know from (11), lagged one period, that

$$n_{t-1}^s \phi^s + n_{t-1}^u \phi^u = \gamma \quad (15)$$

The fertility rate of the economy in period $t - 1$ is

$$n_{t-1} = n_{t-1}^s + n_{t-1}^u$$

At the perfect foresight competitive equilibrium, we have $h_t^e = h_t$. It is straightforward that, the fraction of skilled labor in period t can be rewritten as

$$h_t = \frac{n_{t-1}^s}{n_{t-1}^s + n_{t-1}^u} \quad (16)$$

From (14), (15), and (16) we have

$$\begin{aligned} n_{t-1}^s &= \frac{\gamma h(A_t, L_t)}{(\phi^s - \phi^u)h(A_t, L_t) + \phi^u} \equiv n^s(h(A_t, L_t)) \\ n_{t-1}^u &= \frac{\gamma(1 - h(A_t, L_t))}{(\phi^s - \phi^u)h(A_t, L_t) + \phi^u} \equiv n^u(h(A_t, L_t)) \\ n_{t-1} &= n_{t-1}^s + n_{t-1}^u = \frac{\gamma}{(\phi^s - \phi^u)h(A_t, L_t) + \phi^u} \equiv n(h(A_t, L_t)) \end{aligned}$$

As in (14), when $A_t \leq \hat{A}(L_t)$ then $h(A_t, L_t) = 0$, therefore, in this case

$$n_{t-1}^s = 0$$

$$n_{t-1} = n_{t-1}^u = \frac{\gamma}{\phi^u}$$

and when $A_t > \hat{A}(L_t)$ then

$$\begin{aligned} \frac{\partial n_{t-1}^s}{\partial A_t} &= \frac{\gamma \phi^u}{[(\phi^s - \phi^u)h(A_t, L_t) + \phi^u]^2} \frac{\partial h(A_t, L_t)}{\partial A_t} > 0 \\ \frac{\partial n_{t-1}^u}{\partial A_t} &= \frac{-\gamma \phi^s}{[(\phi^s - \phi^u)h(A_t, L_t) + \phi^u]^2} \frac{\partial h(A_t, L_t)}{\partial A_t} < 0 \\ \frac{\partial n_{t-1}}{\partial A_t} &= \frac{\gamma(\phi^u - \phi^s)}{[(\phi^s - \phi^u)h(A_t, L_t) + \phi^u]^2} \frac{\partial h(A_t, L_t)}{\partial A_t} < 0 \end{aligned}$$

And

$$\lim_{A_t \rightarrow +\infty} n_{t-1} = \lim_{A_t \rightarrow +\infty} n_{t-1}^s = \frac{\gamma}{\phi^s} \quad \text{and} \quad \lim_{A_t \rightarrow +\infty} n_{t-1}^u = 0$$

Figure 7 below presents intuitively the impact of technology on fertility and the structure of labor.

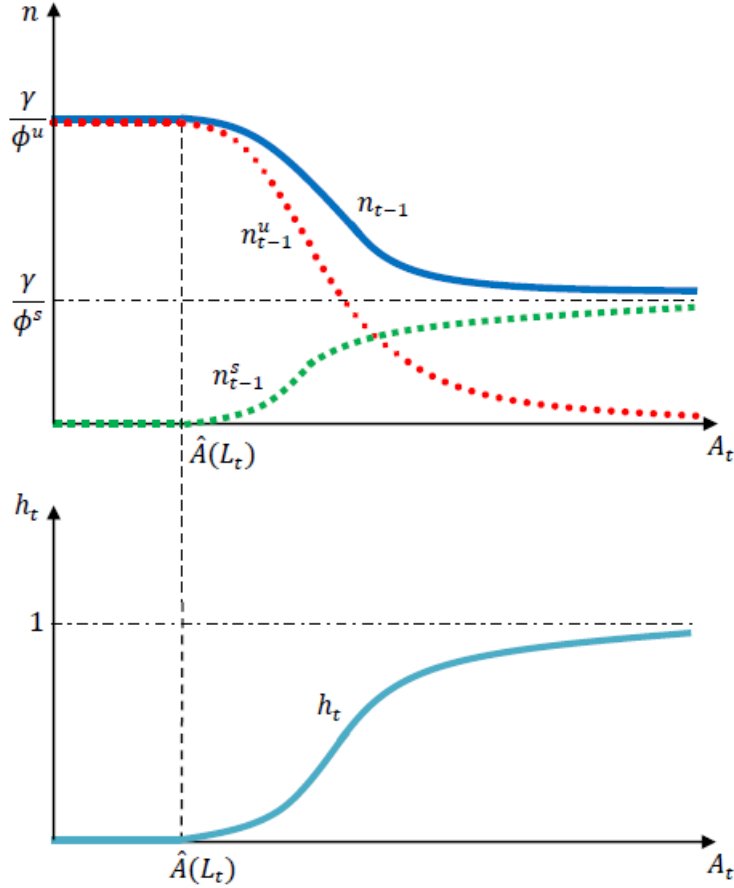


Figure 7. Fertility and structure of labor against technology

6 Competitive equilibrium and Dynamical system

The perfect foresight competitive equilibrium of the economy is characterized by (i) the household utility maximization under the constraints, (ii) the equilibrium in the labor markets, (iii) the dynamics of the technological level, and (iv) the dynamics of population and the structure of labor. Therefore, the perfect foresight competitive equilibrium is a sequence of $\{c_t^i, n_t^s, n_t^u, w_t^s, w_t^u, A_{t+1}, L_{t+1}, h_{t+1}\}_{t \geq 0}$ is determined by the following system of equations

$$c_t^i = (1 - \gamma)w_t^i \quad i \in \{s, u\}$$

$$n_t^s = \frac{\gamma h_{t+1}^e}{(\phi^s - \phi^u)h_{t+1}^e + \phi^u}$$

$$n_t^u = \frac{\gamma(1 - h_{t+1}^e)}{(\phi^s - \phi^u)h_{t+1}^e + \phi^u}$$

$$w_t^s = A_t$$

$$w_t^u = \left(\frac{\chi(A_t)}{L_t(1 - h_t)} \right)^\alpha$$

$$A_{t+1} = [1 + g(h_t)] A_t$$

$$L_{t+1} = \frac{\gamma L_t}{(\phi^s - \phi^u)h_{t+1} + \phi^u}$$

$$h_{t+1}^e = h_{t+1}$$

$$h_{t+1} = \begin{cases} 0 & \text{if } A_{t+1} \leq \hat{A}(L_{t+1}) \\ 1 - \left(\frac{\phi^s}{\phi^u} \right)^{1/\alpha} \frac{\chi(A_{t+1})}{A_{t+1}^{1/\alpha} L_{t+1}} & \text{if } A_{t+1} \geq \hat{A}(L_{t+1}) \end{cases}$$

for given A_0 , L_0 , and h_0 ; where $\hat{A}(L_{t+1})$ is the only solution to (12) given L_{t+1} .

The perfect foresight competitive equilibrium can be fully characterized by the following reduced system describing the equilibrium dynamics of technological level, population size, and structure of labor.

$$A_{t+1} = [1 + g(h_t)] A_t \tag{17}$$

$$L_{t+1} = \frac{\gamma L_t}{(\phi^s - \phi^u)h_{t+1} + \phi^u} \tag{18}$$

$$h_{t+1} = \begin{cases} 0 & \text{if } A_{t+1} \leq \hat{A}(L_{t+1}) \\ 1 - \left(\frac{\phi^s}{\phi^u} \right)^{1/\alpha} \frac{\chi(A_{t+1})}{A_{t+1}^{1/\alpha} L_{t+1}} & \text{if } A_{t+1} \geq \hat{A}(L_{t+1}) \end{cases} \tag{19}$$

for given initial conditions A_0 , L_0 , and h_0 .

This system defines $\{A_{t+1}, L_{t+1}, h_{t+1}\}$ to be a function of its lagged value $\{A_t, L_t, h_t\}$. In effect, when $A_{t+1} \leq \hat{A}(L_{t+1})$ then

$$A_{t+1} = [1 + g(h_t)] A_t$$

$$L_{t+1} = \frac{\gamma L_t}{\phi^u}$$

$$h_{t+1} = 0$$

while when $A_{t+1} \geq \hat{A}(L_{t+1})$, we have

$$A_{t+1} = [1 + g(h_t)] A_t$$

$$L_{t+1} [(\phi^s - \phi^u)h_{t+1} + \phi^u] = \gamma L_t$$

$$L_{t+1}(1 - h_{t+1}) = \left(\frac{\phi^s}{\phi^u} \right)^{1/\alpha} \frac{\chi([1 + g(h_t)] A_t)}{([1 + g(h_t)] A_t)^\alpha}$$

implying that there always exists an equilibrium.

We now study the development process of a country from the agricultural regime transiting to the manufacturing regime (or so-called the industrial regime). Consider an economy in its early stages of development characterized by a sufficiently low level of technology, i.e. $A_0 < \hat{A}_0 = \hat{A}(L_0)$ such that only the agricultural sector is operative. In this regime, fertility is very high and constant at a rate γ/ϕ^u , and the entire labor force consists of unskilled labor, because individuals have no economic incentives to raise skilled offspring, leading to technology grow very slowly at a rate $g(0)$.

Over time, from date t onwards, when technology has accumulated enough, i.e. $A_t \geq \hat{A}_t$, guaranteeing the manufacturing sector to be viable, then individuals raise less unskilled offspring in order to raise more skilled ones, because of their utility maximization behavior, and the increase in the technological level. This makes the structure of labor change towards an increasing fraction of skilled workers in the labor force. Because the cost in time of raising a skilled offspring is higher than that of raising an unskilled one, then the increase in the technological level leads to a decrease in fertility accompanied with an increase in the fraction of skilled worker. The higher fraction of skilled worker, in turn, speeds up the technological progress. This feedback loop between technology and the fraction of skilled worker accelerates the industrialization of the economy characterized by the expansion of the manufacturing sector along with the simultaneous reduction in agricultural sector. Over time, the fertility converges to a constant low rate γ/ϕ^s , the fraction of skilled labor converges to 1.

7 Geography and a Tale of two countries

We define the timing of industrialization to be the time when the manufacturing sector starts to be economically viable, i.e. when it holds

$$A_t/\phi^s = \left(\frac{\chi(A_t)}{L_t} \right)^\alpha / \phi^u$$

This section studies how geography affects the timing and speed of industrialization in the development process of an economy, and it also analyzes a tale of two countries, which differ only in the geography, about the possibility of reversal of fortunes between them. We know from the Proposition 1 that the manufacturing sector is economically viable if, and only if

$$\frac{A_t}{\phi^s} \geq \frac{\left(\frac{\chi(A_t)}{L_t}\right)^\alpha}{\phi^u} \quad \text{i.e.} \quad \frac{A_t}{\chi(A_t)^\alpha} \geq \frac{\phi^s}{\phi^u L_t^\alpha}$$

which implies $A_t \geq \hat{A}(L_t)$.

We say that, country C has more geographical advantage for agriculture production than country B if the following property holds

$$\chi_C(A) > \chi_B(A) \quad \forall A.$$

Proposition 2 below states the effect of geographical advantage for agricultural production on the timing of industrialization.

Proposition 2: *In the overlapping generations economy above, the more geographical advantage for agriculture, the later the timings of industrialization and demographic transition.*

Proof: In effect, let us consider two countries B and C which have the same initial conditions $A_0 > 0$, $L_0 > 0$, and $h_0 = 0$ and are identical except that country C has a geographical advantage for agricultural production over country B , i.e.

$$\chi_C(A_t) > \chi_B(A_t) \quad \forall A_t$$

which implies

$$\frac{A_t}{\chi_B(A_t)^\alpha} > \frac{A_t}{\chi_C(A_t)^\alpha} \quad \forall A_t. \quad (20)$$

Assume that both countries start from the agricultural production regime. As shown in section 4, before the manufacturing sector is operative, i.e. when the level of technology is sufficiently low, the population grows at a constant rate γ/ϕ^u . So the populations of countries B and C are the same during the agricultural development process until at least one of these countries starts having manufacturing production, and hence during this process the growth rates of technological levels in the two countries are the same at a constant $g(0)$. So (20) implies that the manufacturing sector is operative in country B before in country C , i.e. the timing of industrialization in country B precedes that of country C .

The industrialization in country B triggers the decline in fertility along with the change in the labor structure towards an increase in the fraction of skilled labor. Hence, the demographic transition in country B also precedes that of country C .

Q.E.D.

The Proposition 2 shows that the timing of industrialization in a country with less geographical advantage for agricultural production, country B , precedes that of a country with a geographical advantage for agricultural production, country C . This is because during the agricultural development process, the return to labor in agricultural production in country B is always less than that in country C , while the returns to labor in the manufacturing sector, if it were operative, in both countries would be the same due to the same level of technology. So, although in the agricultural development process country C is more prosperous than country B , country C needs a higher level of technology (than country B) to start

industrializing, i.e. country C needs more time for technological accumulation to incentivize individuals to raise skilled offspring. As a result, the timing of industrialization in country B precedes that of country C . Country B enters the industrialization process along with a reduction of the fertility rate, an increase of the fraction of skilled labor, and an accelerated technological progress, while country C continues its agricultural development with a high fertility rate and slow technological progress.

8 Conclusion

This paper develops a unified endogenous growth model to explain the development from agricultural production to manufacturing production along with the demographic transition and an increasing fraction of skilled workers in the labor force. The theoretical results are consistent with the phase of industrial revolution and beyond in Western Europe and Western Offshoots. By taking into account the geographical factor, this paper sheds some light on the fact that a geographical advantage for agricultural production makes a country more prosperous in agricultural regime but delays its industrialization and demographic transition, as well as delays the labor structure change towards an increasing fraction of skilled labor, hence inhibiting technological progress. This may help to explain the reversal in fortunes during the development process between Europe and Asia. This paper points to geography as a fundamental cause of growth and divergence across societies.

This paper builds on Galor and Mountford (2006) and Ashraf and Galor (2012) introducing geographical advantage for agricultural production in a single set-up. This single set-up, however, provides some implications, still absent in both Galor and Mountford (2006) and Ashraf and Galor (2012), for empirical investigations. They are: (i) a high total factor productivity in agriculture delays the demographic transition and industrialization; (ii) fertility, the structure of labor, and growth are all shaped by a geographical advantage for agricultural production.

Of course, geography, particularly geographical advantage for agricultural production, is not everything in explaining the development process and divergence across societies. However, this paper contributes a geographical viewpoint to this literature. Other viewpoints based too on geography, culture, institutions, ethnic, linguistic, religious fractionalization, etc., and also geography to explain the asymmetric development across globe are left for future research, both in theory and empirics.

9 Appendix

A1. Solving the individual's problem

The utility maximization of individual $i \in (s, u)$ in period t is

$$\begin{aligned} \max_{c_t > 0; n_t^{s,i}, n_t^{u,i} \geq 0} \quad & \gamma \ln (w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i}) + (1 - \gamma) \ln c_t^i \\ \text{subject to} \quad & c_t^i + w_t^i (n_t^{s,i} \phi^s + n_t^{u,i} \phi^u) \leq w_t^i \end{aligned}$$

Because the maximization problem is convex, then the Kuhn-Tucker conditions are necessary and sufficient for a maximum. The Kuhn-Tucker conditions of this problem are

$$\begin{pmatrix} \frac{1-\gamma}{c_t^i} \\ \frac{\gamma w_{t+1}^s}{w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i}} \\ \frac{\gamma w_{t+1}^u}{w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i}} \end{pmatrix} = \lambda_1 \begin{pmatrix} 1 \\ w_t^i \phi^s \\ w_t^i \phi^u \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} + \lambda_3 \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$c_t^i + w_t^i (n_t^{s,i} \phi^s + n_t^{u,i} \phi^u) - w_t^i \leq 0$$

$$n_t^{s,i} \geq 0$$

$$n_t^{u,i} \geq 0$$

$$\lambda_1 [c_t^i + w_t^i (n_t^{s,i} \phi^s + n_t^{u,i} \phi^u) - w_t^i] = 0$$

$$\lambda_2 n_t^{s,i} = 0$$

$$\lambda_3 n_t^{u,i} = 0$$

$$\lambda_1, \lambda_2, \lambda_3 \geq 0$$

i.e.

$$\frac{1-\gamma}{c_t^i} = \lambda_1 > 0$$

$$c_t^i + w_t^i (n_t^{s,i} \phi^s + n_t^{u,i} \phi^u) - w_t^i = 0$$

$$n_t^{s,i} \geq 0$$

$$n_t^{u,i} \geq 0$$

$$\frac{\gamma w_{t+1}^s}{w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i}} = \frac{1-\gamma}{c_t^i} w_t^i \phi^s - \lambda_2$$

$$\frac{\gamma w_{t+1}^u}{w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i}} = \frac{1-\gamma}{c_t^i} w_t^i \phi^u - \lambda_3$$

$$\lambda_2 n_t^{s,i} = 0$$

$$\lambda_3 n_t^{u,i} = 0$$

$$\lambda_2, \lambda_3 \geq 0$$

- (i) If at the solution the positivity constraint on skilled children is binding and the one to unskilled children is not binding, i.e. $n_t^{s,i} = 0$, $n_t^{u,i} > 0$, then $\lambda_2 \geq 0$, $\lambda_3 = 0$, and hence

$$\frac{\gamma}{n_t^{u,i}} = \frac{1-\gamma}{c_t^i} w_t^i \phi^u$$

$$c_t^i + w_t^i n_t^{u,i} \phi^u = w_t^i$$

Therefore, we have

$$c_t^i = (1-\gamma)w_t^i$$

$$n_t^{u,i} = \frac{\gamma}{\phi^u}$$

- (ii) If at the solution the positivity constraint on skilled children is not binding and the one on unskilled children is binding, i.e. $n_t^{s,i} > 0$, $n_t^{u,i} = 0$, then $\lambda_2 = 0$, $\lambda_3 \geq 0$, and

$$\frac{\gamma}{n_t^{s,i}} = \frac{1-\gamma}{c_t^i} w_t^i \phi^s$$

$$c_t^i + w_t^i n_t^{s,i} \phi^s = w_t^i$$

Therefore, we have

$$c_t^i = (1-\gamma)w_t^i$$

$$n_t^{s,i} = \frac{\gamma}{\phi^s}$$

- (iii) If solution is interior, i.e. $n_t^{s,i} > 0$, $n_t^{u,i} > 0$, then $\lambda_2 = 0$, $\lambda_3 = 0$, and hence

$$\frac{\gamma w_{t+1}^s}{w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i}} = \frac{1-\gamma}{c_t^i} w_t^i \phi^s$$

$$\frac{\gamma w_{t+1}^u}{w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i}} = \frac{1-\gamma}{c_t^i} w_t^i \phi^u$$

which imply that it must hold

$$\frac{w_{t+1}^s}{w_{t+1}^u} = \frac{\phi^s}{\phi^u}$$

Therefore,

$$\begin{aligned} \frac{\gamma w_{t+1}^u}{w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i}} &= \frac{1-\gamma}{c_t^i} w_t^i \phi^u \quad \Leftrightarrow \quad \frac{\gamma}{\frac{w_{t+1}^s}{w_{t+1}^u} n_t^{s,i} + n_t^{u,i}} = \frac{1-\gamma}{c_t^i} w_t^i \phi^u \\ \Leftrightarrow \quad \frac{\gamma}{n_t^{s,i} \phi^s + n_t^{u,i} \phi^u} &= \frac{1-\gamma}{c_t^i} w_t^i \end{aligned} \quad (21)$$

We also have

$$c_t^i + w_t^i (n_t^{s,i} \phi^s + n_t^{u,i} \phi^u) = w_t^i \quad (22)$$

then from (21) and (22) we have

$$c_t^i = (1-\gamma) w_t^i$$

$$n_t^{s,i} \phi^s + n_t^{u,i} \phi^u = \gamma$$

So, in all cases (i), (ii), and (iii), we have

$$c_t^i = (1-\gamma) w_t^i$$

$$n_t^{s,i} \phi^s + n_t^{u,i} \phi^u = \gamma$$

Now, we will clarify under what conditions each corresponding case occurs. We know from the two last equations that the allocation of resources between raising children and consumption is always constant. The last equation shows that the total time for raising children is also always constant. So the optimization problem of individuals boils down to determine the optimal choice between skilled and unskilled children so as to maximize their total income, $w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i}$, under the constraint $n_t^{s,i} \phi^s + n_t^{u,i} \phi^u = \gamma$.

From the last equation we have

$$0 \leq n_t^{s,i} \leq \frac{\gamma}{\phi^s}$$

$$0 \leq n_t^{u,i} \leq \frac{\gamma}{\phi^u}$$

We have

$$\begin{aligned} w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i} &= w_{t+1}^s n_t^{s,i} + w_{t+1}^u \frac{\gamma - n_t^{s,i} \phi^s}{\phi^u} \\ &= \left(w_{t+1}^s - w_{t+1}^u \frac{\phi^s}{\phi^u} \right) n_t^{s,i} + w_{t+1}^u \frac{\gamma}{\phi^u} \end{aligned}$$

- If $\frac{w_{t+1}^s}{\phi^s} < \frac{w_{t+1}^u}{\phi^u}$, then $(w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i})_{\max} = w_{t+1}^u \frac{\gamma}{\phi^u}$ when $n_t^{s,i} = 0$ and $n_t^{u,i} = \frac{\gamma}{\phi^u}$.
- If $\frac{w_{t+1}^s}{\phi^s} > \frac{w_{t+1}^u}{\phi^u}$, then $(w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i})_{\max} = \left(w_{t+1}^s - w_{t+1}^u \frac{\phi^s}{\phi^u} \right) \frac{\gamma}{\phi^s}$ when $n_t^{s,i} = \frac{\gamma}{\phi^s}$ and $n_t^{u,i} = 0$.
- If $\frac{w_{t+1}^s}{\phi^s} = \frac{w_{t+1}^u}{\phi^u}$, then any composition satisfying $n_t^{s,i} \phi^s + n_t^{u,i} \phi^u = \gamma$ is an optimal solution.

In summary, we have

$$\begin{aligned} n_t^{s,i} = 0, n_t^{u,i} = \frac{\gamma}{\phi^u} & \quad \text{if} \quad \frac{w_{t+1}^s}{\phi^s} < \frac{w_{t+1}^u}{\phi^u} \\ n_t^{s,i} = \frac{\gamma}{\phi^s}, n_t^{u,i} = 0 & \quad \text{if} \quad \frac{w_{t+1}^s}{\phi^s} > \frac{w_{t+1}^u}{\phi^u} \\ n_t^{s,i} > 0, n_t^{u,i} > 0 & \quad \text{only if} \quad \frac{w_{t+1}^s}{\phi^s} = \frac{w_{t+1}^u}{\phi^u}. \end{aligned}$$

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